Cvičenie — 1. týždeň

1. V obore komplexných čísel riešte rovnice

a) $2z + 3iz - 1 + 2i = -2 + i$ $\quad [-\frac{5}{13} + i\frac{1}{13}]$, $\qquad$ b) $\dfrac{1+i}{2z+iz-i} = \dfrac{1}{z+1-i}$, $\quad [2+i]$,

c) $\frac{1}{z+i} - \frac{1+i}{z} = 1$ $\quad \left[z_1 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}-2}{2}i,\ z_2 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}+2}{2}i\right]$.

2. Riešte binomické rovnice, riešenie znázornite v komplexnej rovine.
a) $z^2 = i$, $\quad \left[z_k = e^{i(\frac{\pi}{4}+k\pi)},\ k = 0,1\right]$; $\qquad$ b) $z^4 = -4$, $\quad \left[z_k = \sqrt{2}e^{i(\frac{\pi}{4}+k\frac{\pi}{2})},\ k = 0,1,2,3\right]$

c) $z^4 = -2 + 2\sqrt{3}i$, $\quad \left[z_k = \sqrt{2}e^{i(\frac{\pi}{6}+k\frac{\pi}{2})},\ k = 0,1,2,3\right]$; $\qquad$ d) $z^3 = -i$, $\quad \left[z_k = e^{i(\frac{\pi}{2}+k\frac{2\pi}{3})},\ k = 0,1,2\right]$.

3. Vypočítajte čísla a) $\left(\dfrac{1+i}{1-i}\right)^{2016}$, $\quad [1]$; $\quad$ b) $\dfrac{(1-i)(1-2i)}{3+4i}$, $\quad [-\frac{3}{5} - \frac{1}{5}i]$ $\quad$ a znázornite ich v komplexnej rovine.

4a. Určte $p \in \{0, 1, \ldots, 16\}$ a $q \in N$, pre ktoré je $291 = 17q + p$ $\quad [p = 2,\ q = 17]$.

4b. Určte $a \in Z_{17}$, pre ktoré je $291 \equiv a \pmod{17}$ $\quad [2]$

5. Určte mnohočlen $p(x) \in P(R)$ stupňa najviac 2, a mnohočlen $q(x) \in P(R)$, pre ktoré platí
$2x^4 - x^3 + x + 2 = (x^3 - x^2 + 1)q(x) + p(x)$. $\quad [q(x) = 2x + 1,\ p(x) = x^2 - x + 1]$

6. Zistite, či je množina $F = \{x + \sqrt{2}y \colon x, y \in Q\}$ s obvyklými operáciami pole.