

Testovanie hypotéz I

Príklad.

$X \sim A(p)$, p je neznámy parameter

Jednoduchá hypotéza proti jednoduchej alternatíve:

$$H_0 : p = 0.2$$

$$H_1 : p = 0.5$$

Testovacia štatistika: $T(X_1, X_2, X_3) = \sum_{i=1}^3 X_i$

Kritická oblasť:

$$W_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : T(x_1, x_2, x_3) \geq 2\}$$

$$\alpha_1 = P(T(X_1, X_2, X_3) \in W_1 \mid H_0) = 0.104$$

$$\beta_1 = P(T(X_1, X_2, X_3) \notin W_1 \mid H_1) = 0.5$$

$$B = \text{pdf}(\text{Bi}(i,3,j/10)) =$$

1.0000	0	0	0
0.7290	0.2430	0.0270	0.0010
0.5120	0.3840	0.0960	0.0080
0.3430	0.4410	0.1890	0.0270
0.2160	0.4320	0.2880	0.0640
0.1250	0.3750	0.3750	0.1250
0.0640	0.2880	0.4320	0.2160
0.0270	0.1890	0.4410	0.3430
0.0080	0.0960	0.3840	0.5120
0.0010	0.0270	0.2430	0.7290
0	0	0	1.0000

$$W_2 = \{(x_1, x_2, x_3) \in \mathbb{R}^3: T(x_1, x_2, x_3) \geq 1\}$$

$$\alpha_2 = P(T(X_1, X_2, X_3) \in W_2 \mid H_0) = 0.488$$

$$\beta_2 = P(T(X_1, X_2, X_3) \notin W_2 \mid H_1) = 0.125$$

$$W_3 = \{(x_1, x_2, x_3) \in \mathbb{R}^3: T(x_1, x_2, x_3) \geq 3\}$$

$$\alpha_3 = P(T(X_1, X_2, X_3) \in W_3 \mid H_0) = 0.008$$

$$\beta_3 = P(T(X_1, X_2, X_3) \notin W_3 \mid H_1) = 0.875$$

Zložená hypotéza proti zloženej alternatíve:

$$H_0 : p \leq 0.2$$

$$H_1 : p > 0.2$$

$$W = W_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3: T(x_1, x_2, x_3) \geq 2\}$$

Silofunkcia testu:

$$\begin{aligned} 1 - \beta(p) &= 1 - P(T(X_1, X_2, X_3) \notin W \mid p) = \\ &= P(T(X_1, X_2, X_3) \in W \mid p) \end{aligned}$$

