

(1) Riešte okrajovú úlohu

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < 2, \quad 0 < y < 2,$$

$$u(0, y) = u(2, y) = u(x, 0) = 0, \quad \frac{\partial u}{\partial y}(x, 2) = 1.$$

$$\underline{u(x, y) = \sum_{k=1}^{\infty} \frac{4}{(k\pi)^2} \frac{1-(-1)^k}{\cosh(k\pi)} \sinh \frac{k\pi}{2} x \sin k \frac{\pi}{2} x.}$$

(2) Riešte okrajovú úlohu

$$\Delta u(x, y) = 1, \quad 1 < x^2 + y^2 < 4, \quad u(x, y) = 0, \quad \text{ak } x^2 + y^2 = 1,$$

$$\frac{\partial u}{\partial n} = 0, \quad \text{ak } x^2 + y^2 = 4.$$

$$\underline{u(x, y) = u(r) = \frac{1}{4}r^2 - 2 \ln r - \frac{1}{4}.}$$

(3) Riešte začiatočno-okrajovú úlohu

$$\frac{\partial u}{\partial t} - 4 \frac{\partial^2 u}{\partial x^2} = 0, \quad t > 0, \quad 0 < x < \pi,$$

$$u(0, x) = x, \quad \frac{\partial u}{\partial x}(t, 0) = u(t, \pi) = 0.$$

$$\underline{u(t, x) = \sum_{k=1}^{\infty} \left(\frac{-8}{\pi(2k-1)^2} + \frac{4(-1)^{k-1}}{2k-1} \right) e^{-(2k-1)^2 t} \cos\left(\frac{2k-1}{2}x\right).}$$