(1) (5b) Riešte okrajovú úlohu $-(1+x^2)u'' - 2xu' = 2,\ u(0) = u'(1) = 0$

$-((1+x^2)u')' = 2,\ ((1+x^2)u')' = -2,$
$(1+x^2)u' = -2x + c_1,\ u' = \frac{-2x+c_1}{1+x^2}$

$u(x) = \int \frac{-2x+c_1}{1+x^2} dx = -\ln(1+x^2) + c_1 \arctan x + c_2,$

$u(0) = c_2 = 0,\ u'(1) = \frac{-2+c_1}{2} = 0 \Rightarrow c_1 = 2,$

$u(x) = 2\arctan x - \ln(1+x^2),\ 0 < x < 1.$

(2) (5b) Riešte okrajovú úlohu $-u'' + 4u = 0,\ u(0) = 1,\ u(2) + u'(2) = 0$

$u(x) = c_1 \cosh 2x + c_2 \sinh 2x.$
$u(0) = c_1 = 1,$
$u(2) + u'(2) = \cosh 4 + c_2 \sinh 4 + 2\sinh 4 + c_2 \cosh 4 = 0,$
$c_2 = -\frac{\cosh 4 + 2\sinh 4}{\sinh 4 + 2\cosh 4},$

$u(x) = \cosh 2x - \frac{\cosh 4 + 2\sinh 4}{\sinh 4 + 2\cosh 4} \sinh 2x$

$= \frac{1}{3e^4 + e^{-4}}(e^{2x-4} + 3e^{4-2x}).$

(3) (6b) Riešte Sturmovu-Liovilleovu úlohu $u'' + \lambda u = 0,\ 0 < x < 2\pi,$

a) $u'(0) = u'(2\pi) = 0.$

$\lambda \geq 0.$
$\lambda = 0 \Rightarrow u'' = 0 \Rightarrow u(x) = c_1 x + c_2.$
$u'(0) = u'(2\pi) = c_1 = 0,\ c_2 = 1,$
$\lambda_0 = 0,\ u_0(x) = 1,$

$\lambda > 0 \Rightarrow u(x) = c_1 \cos\sqrt{\lambda}x + c_2 \sin\sqrt{\lambda}x,$
$u'(x) = -c_1\sqrt{\lambda}\sin\sqrt{\lambda}x + c_2\sqrt{\lambda}\cos\sqrt{\lambda}x,$
$u'(0) = c_2\sqrt{\lambda} = 0 \Rightarrow c_2 = 0,\ c_1 = 1$
$u'(2\pi) = \sqrt{\lambda}\sin\sqrt{\lambda}2\pi = 0 \Rightarrow \sqrt{\lambda}2\pi = n\pi,\ n = 1, 2, ...$

$\lambda_n = \frac{n^2}{4},\ u_n(x) = \cos\frac{n}{2}x,\ n = 0, 1, 2, ...$

b) $u(0) = u'(2\pi) = 0.$

$\lambda > 0 \Rightarrow u(x) = c_1 \cos\sqrt{\lambda}x + c_2 \sin\sqrt{\lambda}x,$
$u'(x) = -c_1\sqrt{\lambda}\sin\sqrt{\lambda}x + c_2\sqrt{\lambda}\cos\sqrt{\lambda}x,$
$u(0) = c_1 = 0,\ c_2 = 1$
$u'(2\pi) = \sqrt{\lambda}\cos\sqrt{\lambda}2\pi = 0 \Rightarrow \sqrt{\lambda}2\pi = (2n-1)\frac{\pi}{2},$

$\lambda_n = \frac{2n-1)^2}{16},\ u_n(x) = \sin\frac{2n-1}{4}x,\ n = 1, 2, ...$