

Príklady:

1. Riešte začiatovo-okrajovú úlohu $u_t - 9u_{xx} = 0, t > 0$,
 - a) $0 < x < \frac{\pi}{2}, u(t, 0) = u_x(t, \frac{\pi}{2}) = 0, u(0, x) = x$
 - b) $0 < x < 1, u_x(t, 0) = u(t, 1) = 0, u(0, x) = 1,$
 - c) $0 < x < \pi, u_x(t, 0) = u_x(t, \pi) = 0, u(0, x) = \sin x$
 - d) $0 < x < 2, u(t, 0) = u(t, 2) = 0, u(0, x) = x^2.$
2. Riešte začiatovo-okrajovú úlohu $u_t - a^2(u_{xx} + u_{yy}) = 0, t > 0$,
 - a) $0 < x < 1, 0 < y < 1, u(t, 0, y) = u(t, 1, y) = u_y(t, x, 0) = u(t, x, 1) = 0, u(0, x, y) = -1,$
 - b) $0 < x < 2, 0 < y < 1, u(t, 0, y) = u(t, 2, y) = u(t, x, 0) = u(t, x, 1) = 0, u(0, x, y) = xy,$
 - c) $0 < x < \pi, 0 < y < \pi, u(t, 0, y) = u(t, \pi, y) = u_y(t, x, 0) = u_y(t, x, \pi) = 0,$
 $u(0, x, y) = x(\pi - y).$
3. Riešte začiatovo-okrajovú úlohu $u_t - 9u_{xx} = f(t, x), t > 0$,
 - a) $0 < x < \frac{\pi}{2}, u(t, 0) = u_x(t, \frac{\pi}{2}) = 0, u(0, x) = x, f(t, x) = At$
 - b) $0 < x < 1, u_x(t, 0) = u(t, 1) = 0, u(0, x) = 1, f(t, x) = Ae^{-t}$
 - c) $0 < x < \pi, u_x(t, 0) = u_x(t, \pi) = 0, u(0, x) = \sin x, f(t, x) = t \sin x$
 - d) $0 < x < 2, u(t, 0) = u(t, 2) = 0, u(0, x) = x^2, f(t, x) = A\delta_1,$
kde δ_1 je Diracova distribúcia v bode $x = 1$, pre ktorú (symbolicky)
 $\int_0^2 \delta_1(x)\varphi(x)dx = \varphi(1)$ pre každú spojitú funkciu $\varphi : [0, 2] \rightarrow R$.
4. Riešte začiatovo-okrajovú úlohu $u_t - a^2(u_{xx} + u_{yy}) = f(t, x, y), t > 0$,
 - a) $0 < x < 1, 0 < y < 1, u(t, 0, y) = u(t, 1, y) = u_y(t, x, 0) = u(t, x, 1) = 0, u(0, x, y) = -1,$
 $f(t, x, y) = tx,$
 - b) $0 < x < 2, 0 < y < 1, u(t, 0, y) = u(t, 2, y) = u(t, x, 0) = u(t, x, 1) = 0, u(0, x, y) = xy,$
 $f(t, x, y) = Ae^{-2t},$
 - c) $0 < x < \pi, 0 < y < \pi, u(t, 0, y) = u(t, \pi, y) = u_y(t, x, 0) = u_y(t, x, \pi) = 0,$
 $u(0, x, y) = x(\pi - y), f(t, x, y) = At^2.$

V nehomogénnych úlohách 3 resp. 4 čiastočne použite výsledky úloh 1 resp. 2. Vlastné hodnoty operátora kL , $k \in R$ majú tvar $\lambda_n = k\mu_n$, kde μ_n , $n = 1, 2, \dots$ sú vlastné hodnoty operátora L . Vlastné funkcie sa nemenia.