Boundary Value Problems for Ordinary Differential Equations

1. Solve the boundary value problem $\quad -u'' + u = f(x)$,

   a) $0 < x < 2,\ f(x) = 1,\ u(0) = 0,\ (2) = -1,$

   b) $0 < x < 2,\ f(x) = 1,\ u'(0) = 0,\ u'(2) + u(2) = 0,$

   c) $0 < x < 1,\ f(x) = e^{2x},\ u(0) = 1,\ u(2) = 2$

   d) $0 < x < 1,\ f(x) = e^{-x},\ u(0) = 0,\ u'(2) = 0.$

2. Solve the boundary value problem after expressing the equation into the form $(p(x)u')' = f(x)$:

   $xu'' + u' = f(x),$

   a) $1 < x < 2,\ f(x) = x,\ u'(1) = 0,\ u(2) = 0,$

   b) $1 < x < 2,\ f(x) = x,\ u'(1) - u(1) = 0,\ u'(2) = 0,$

   c) $1 < x < 3,\ f(x) = 2 - x,\ u'(1) = u'(3) = 0$ (infinitely many solutions)

3. Solve the boundary value problem

   $x^2u'' + 2xu' = f(x),$

   a) $1 < x < 2,\ f(x) = 1,\ u(1) = 0,\ u'(2) = -1,$

   b) $1 < x < 2,\ f(x) = x,\ u'(1) - u(1) = 0,\ u'(2) = 0,$

4. Solve the Sturm-Lioville problem for eigenvalues and eigenfunctions

   $u'' + \lambda u = 0,$ with boundary conditions

   a) $0 < x < 4,\ u(0) = u(4) = 0,$

   b)$0 < x < 4,\ u'(0) = u'(4) = 0,$

   c)$0 < x < 4,\ u(0) = u'(4) = 0,$

   d) $0 < x < 4,\ u'(0) = u(4) = 0,$

   e) $0 < x < \frac{\pi}{2},\ u(0) = u(\frac{\pi}{2}) = 0,$

   f) $0 < x < 1,\ u(0) - u'(0) = u(1) = 0$