Príklady:

1. Riešte začiatočno-okrajovú úlohu $u_{tt} - 9u_{xx} = 0,\ t > 0,$

   a) $0 < x < \frac{\pi}{2},\ u(t,0) = \ u_x(t,\frac{\pi}{2}) = 0,\ u(0,x) = x,\ u_t(0,x) = 1$

   b) $0 < x < 1,\ u_x(t,0) = u(t,1) = 0,\ u(0,x) = 1, u_t(0,x) = x$

   c) $0 < x < \pi,\ u_x(t,0) = u_x(t,\pi) = 0,\ u(0,x)) = \sin x,\ u_t(0,x) = 0$

   d) $0 < x < 2,\ u(t,0) = u(t,2) = 0,\ u(0,x) = 0,\ u_t(0,x) = 1.$

2. Riešte začiatočno-okrajovú úlohu $u_{tt} - a^2(u_{xx} + u_{yy}) = 0,\ t > 0,$

   a) $0 < x < 1,\ 0 < y < 1,\ u(t,0,y) = u(t,1,y) = u_y(t,x,0) = u(t,x,1) = 0,\ u(0,x,y) = -1, u_t(0,x,y) = 1$

   b) $0 < x < 2,\ 0 < y < 1,\ u(t,0,y) = u(t,2,y) = u(t,x,0) = u(t,x,1) = 0,\ u(0,x,y) = xy, u_t(0,x,y) = 0$

   c) $0 < x < \pi,\ 0 < y < \pi,\ u(t,0,y) = u(t,\pi,y) = u_y(t,x,0) = u_y(t,x,\pi) = 0,$
   $u(0,x,y) = x(\pi - y),\ u_t(0,x,y) = 2.$

3. Riešte začiatočno-okrajovú úlohu $u_{tt} - 9u_{xx} = f(t,x),\ t > 0,$

   a) $0 < x < \frac{\pi}{2},\ u(t,0) = \ u_x(t,\frac{\pi}{2}) = 0,\ u(0,x) = x,\ u_t(0,x) = 1,\ f(t,x) = At$

   b) $0 < x < 1,\ u_x(t,0) = u(t,1) = 0,\ u(0,x) = 1,\ u_t(0,x) = x,\ f(t,x) = Ae^{-t}$

   c) $0 < x < \pi,\ u_x(t,0) = u_x(t,\pi) = 0,\ u(0,x)) = \sin x,\ u_t(0,x) = 1,\ f(t,x) = t\sin x$

   d) $0 < x < 2,\ u(t,0) = u(t,2) = 0,\ u(0,x) = 0,\ u_t(0,x) = 1,\ f(t,x) = A\delta_1,$

   kde δ_1 je Diracova distribúcia v bode $x = 1$, pre ktorú (symbolicky)
   $\int_0^2 \delta_1(x)\varphi(x)dx = \varphi(1)$ pre každú spojitú funkciu $\varphi : [0,2] \to R.$

4. Riešte začiatočno-okrajovú úlohu $u_{tt} - a^2(u_{xx} + u_{yy}) = f(t,x,y),\ t > 0,$

   a) $0 < x < 1,\ 0 < y < 1,\ u(t,0,y) = u(t,1,y) = u_y(t,x,0) = u(t,x,1) = 0,$
   $u(0,x,y) = -1,\ u_t(0,x,y) = 1,\ f(t,x,y) = tx,$

   b) $0 < x < 2,\ 0 < y < 1,\ u(t,0,y) = u(t,2,y) = u(t,x,0) = u(t,x,1) = 0,$
   $u(0,x,y) = xy,\ u_t(0,x,y) = 0,\ f(t,x,y) = Ae^{-2t},$

   c) $0 < x < \pi,\ 0 < y < \pi,\ u(t,0,y) = u(t,\pi,y) = u_y(t,x,0) = u_y(t,x,\pi) = 0,$
   $u(0,x,y) = x(\pi - y),\ u_t(0,x,y) = 2\ f(t,x,y) = At^2.$

V nehomogénnych úlohách 3 resp. 4 čiastočne použite výsledky úloh 1 resp. 2. Vlastné hodnoty operátora $k\,L,\ k \in R$ majú tvar $\lambda_n = k\mu_n$, kde $\mu_n,\ n = 1,2,...$ sú vlastné hodnoty operátora L. Vlastné funkcie sa nemenia.