Ditkin's condition and ideals with at most countable hull in algebras of functions analytic in the unit disc

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The talk is based on a joint work with

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We denote by $\mathbb{T}$ the unit circle and by $\mathbb{D}$ the unit disc. Let $\mathcal{B}$ be a Banach algebra with respect to the norm $\|\cdot\|_{\mathcal{B}}$ which is a subalgebra of the classical disc algebra $A(\mathbb{D})$ and satifies the following conditions:

(H1) The space of polynomials is a dense subset of $\mathcal{B}$.

(H2) $\lim_{n\to\infty} \|\alpha^n\|_{\mathcal{B}}^{\frac{1}{n}} = 1 \qquad (\alpha \text{ denotes the identity function } z \mapsto z).$

(H3) There exist $k \geq 0$ and $C > 0$ such that

$$|1-|\lambda||^k \|f\|_{\mathcal{B}} \leq C\|(\alpha-\lambda)f\|_{\mathcal{B}}, \quad f \in \mathcal{B},\ |\lambda| < 2.$$

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(H3) There exist $k \geq 0$ and $C > 0$ such that

$$|1-|\lambda||^k \|f\|_{\mathcal{B}} \leq C\|(\alpha-\lambda)f\|_{\mathcal{B}}, \quad f \in \mathcal{B},\ |\lambda| < 2.$$

Remark

We shall assume in the sequel that any Banach algebra $\mathcal{B}$ has a unit $1_{\mathcal{B}} = 1$ and that $\|1\|_{\mathcal{B}} = 1$. Since the algebra $A(\mathbb{D})$ is semisimple the embedding $\mathcal{B}$ into $A(\mathbb{D})$ is continuous. Conditions (H1) and (H2) imply that the maximal ideal space of the algebra $\mathcal{B}$, $\mathfrak{M}(\mathcal{B})$, can be identified with $\overline{\mathbb{D}}$ via the mapping $z \mapsto \delta_z$, where $\delta_z(f) = f(z)$. From this it follows that the algebra $\mathcal{B}$ is semisimple.

Agrafeuil and Zarrabi (2008) studied a structure of closed ideals in algebras satisfying conditions (H1) – (H3). Earlier Faĭvyševskiĭ (1973 – 74) also considered the same problem in algebras satisfying similar assumptions.

In all mentioned cases it is assumed (in a more or less explicite form) that a considered algebra $\mathcal{B}$ is embedded in the algebra $A^{(N_{\mathcal{B}})}(\mathbb{D})$ of functions analytic in $\mathbb{D}$ (with pointwise multiplication) and of class $C^{(N_{\mathcal{B}})}$ on the closed disc $\overline{\mathbb{D}}$ for some $N_{\mathcal{B}} \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

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Moreover, it is assumed that the algebra $\mathcal{B}$ satisfies the analytic Ditkin condition which says the following:

for every point z_0 in the unit circle $\mathbb{T}$ and for every function f from the algebra which satisfies $f^{(j)}(z_0) = 0$ for $0 \leq j \leq N_{\mathcal{B}}$, there exists in $\mathcal{B}$ a sequence (σ_n) such that $\sigma_n(z_0) = 0$ all n and $\lim_{n\to\infty} \|\sigma_n f - f\|_{\mathcal{B}} = 0$.

If U is an inner function and $f \in \mathcal{B}$ the symbol $U|f$ means that U divides f, i.e. there exits a function $\varphi \in H^\infty$ such that $f = U\varphi$.

We denote by U_I the greatest common inner divisor of all nonzero functions in I and we set

$$h^j(I) = \{z \in \mathbb{T} \colon\ f(z) = f'(z) = \ldots = f^{(j)}(z) = 0 \text{ for all } f \in I\}.$$

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A closed ideal I of the algebra $\mathcal{B}$ is standard, if

$$I = \{f \in \mathcal{B} \colon \ U_I|f \text{ and } f^{(j)}(z) = 0 \text{ for } z \in h^j(I), \ 0 \le j \le N_{\mathcal{B}}\}.$$

The following theorem is the principal result of the paper of Agrafeuil and Zarrabi:

Theorem (AZ)

Let $\mathcal{B}$ be a subalgebra of the algebra $A(\mathbb{D})$ which is a Banach algebra endowed with the norm $\|\cdot\|_{\mathcal{B}}$. Suppose that $\mathcal{B}$ satisfies conditions (H1)–(H3) *and the analytic Ditkin condition. Let I be a closed ideal in $\mathcal{B}$ such that the hull*

$$h(I) = \{z \in \overline{\mathbb{D}} \colon\ f(z) = 0 \text{ for } f \in I\}$$

is at most countable. Then the ideal I is standard.

The analytic Ditkin condition is a very strong assumption which confines applicability of the theorem (AZ). We show a simple example of a Banach algebra $\mathcal{B}$ of analytic functions in the unit disc for which $N_{\mathcal{B}} = 0$ and in which Ditkin's condition does not hold. Therefore theorem (AZ) cannot be applied to describe the structure of closed ideals with at most countable hull in that algebra. On the other hand, a form of the closed ideals with at most countable hull in that algebra is known and all such ideals are standard.

Example

Let $\mathcal{A}_1^{(1)}(\mathbb{D})$ be the space of functions f on $\overline{\mathbb{D}}$ which are

(1) continuous on $\overline{\mathbb{D}}$, analytic on $\mathbb{D}$,

(2) of class $C^{(1)}$ on $\overline{\mathbb{D}} \setminus \{1\}$,

(3) satisfy $\lim_{z\to 1}(1-z)f'(z) = 0$.

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We endowe the space $\mathcal{A}_1^{(1)}(\mathbb{D})$ with the norm

$$\|f\| = \sup_{z\in\overline{\mathbb{D}}} |f(z)| + \sup_{z\in\overline{\mathbb{D}}\setminus\{1\}} |(1-z)f'(z)|.$$

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By the maximum principle it follows that this norm coincides on $\mathcal{A}_1^{(1)}(\mathbb{D})$ with the norm

$$\|f\|_{\mathbb{T}} = \sup_{z\in\mathbb{T}} |f(z)| + \sup_{z\in\mathbb{T}\setminus\{1\}} |(1-z)f'(z)|.$$

It is easy to verify that the latter norm is submultiplicative for all functions on $\mathbb{T}$ for which the right hand side of the formula makes sense.

It is clear that the space $\mathcal{A}_1^{(1)}(\mathbb{D})$ is a unital algebra with respect to the pointwise multiplication. Moreover it is complete with respect to the norm $\|\cdot\|_{\mathbb{T}}$. Hence $\mathcal{A}_1^{(1)}(\mathbb{D})$ is a Banach algebra continuously embedded in the disc algebra $A(\mathbb{D})$.

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The algebra $\mathcal{A}_1^{(1)}(\mathbb{D})$ is only a very simple example of an algebra which contains functions having certain properties of differentiability at different boundary points of the unit disc. Algebras of this form appear in a natural way as images of Gelfand transforms of convolution algebras of the Sobolev type.

We need the following properties of this algebra.

Proposition

(i) *The space of polynomials is dense in* $\mathcal{A}_1^{(1)}(\mathbb{D})$.

(ii) $\lim_{n\to\infty} \|\alpha^n\|^{\frac{1}{n}} = 1$.

(iii) $|1-|\lambda||^2\|f\| \leq 3\|(\alpha-\lambda)f\|$ *for all* $f \in \mathcal{A}_1^{(1)}(\mathbb{D})$, $|\lambda| < 2$.

Proof.

(i) Galé and Wawrzyńczyk has proved that the space $\mathcal{A}_1^{(1)}(\mathbb{D})$ is isomorphic under the mapping $f \mapsto (1-\alpha)f$ with a closed ideal in the algebra $A^{(1)}(\mathbb{D})$ of functions analytic in $\mathbb{D}$ and of the class $C^{(1)}$ on the closed disc. More exactly

$$(1-\alpha)\mathcal{A}_1^{(1)}(\mathbb{D}) = I_1 = \{g \in A^{(1)}(\mathbb{D}) \colon\ g(1) = 0\}.$$

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(ii) By definition of the norm

$$\|\alpha^n\| = \|\alpha^n\|_{\mathbb{T}} = 1 + n \sup_{z \in \mathbb{T} \setminus \{1\}} |(1-z)z^{n-1}| = 2n+1,$$

which gives (ii).

(iii) For $|\lambda| = 1$ the inequality in (iii) is obvious. Let $|\lambda| \neq 1$. We have

$$\|f\| = \|f\|_{\mathbb{T}} = \left\|(\alpha - \lambda) f \frac{1}{\alpha - \lambda}\right\|_{\mathbb{T}} \leq \|(\alpha - \lambda) f\|_{\mathbb{T}} \left\|\frac{1}{\alpha - \lambda}\right\|_{\mathbb{T}}.$$

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Since

$$\begin{aligned}\left\|\frac{1}{\alpha - \lambda}\right\|_{\mathbb{T}} &= \sup_{z \in \mathbb{T}} \left|\frac{1}{z - \lambda}\right| + \sup_{z \in \mathbb{T} \setminus \{1\}} \left|\frac{1 - z}{(z - \lambda)^2}\right| \leq \frac{1}{|1 - |\lambda||} + \frac{2}{|1 - |\lambda||^2} \\ &\leq \frac{3}{|1 - |\lambda||^2}\end{aligned}$$

property (iii) is proved.

Hence the algebra $\mathcal{A}_1^{(1)}(\mathbb{D})$ satisfies conditions (H1), (H2), and (H3). The number $N_{\mathcal{B}} = 0$ for $\mathcal{B} = \mathcal{A}_1^{(1)}(\mathbb{D})$ since there are elements of $\mathcal{A}_1^{(1)}(\mathbb{D})$ which are not derivable at 1 (for example the functions $\varphi_n(z) = (1-z)^{\frac{1}{n}}$ defined by an appropiate branch of logarithm).

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The set

$$I_{-1}^{1} = \{f \in \mathcal{A}_1^{(1)}(\mathbb{D}) \colon \ f(-1) = f'(-1) = 0\}$$

is obviously a closed ideal in $\mathcal{A}_1^{(1)}(\mathbb{D})$ with the one-point hull $h(I_{-1}^{1}) = \{-1\}$. However it does not have a standard form from Theorem (AZ).

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We see that in this case Theorem (AZ) does not work. The reason for that is that the algebra $\mathcal{A}_1^{(1)}(\mathbb{D})$ does not satisfy the analytic Ditkin condition.

We have proved that under a modified Ditkin's condition and suitably extended definition of a standard ideal the analogous result to theorem (AZ) holds true.

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We introduce the following two conditions:

(S) For every $z_0 \in \mathbb{T}$ there exists $N(z_0) \in \mathbb{N}_0$, the maximal number amongst $j \in \mathbb{N}_0$ for which functionals $\mathcal{B} \ni f \mapsto f^{(j)}(z_0)$ are well-defined and continuous.

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(D) For every $z_0 \in \mathbb{T}$ there exists a sequence (φ_n) in the algebra $\mathcal{B}$ such that $\varphi_n(z_0) = 0$ for all n and

$$\lim_{n\to\infty} \|(\alpha - z_0)^{N(z_0)+1}\varphi_n - (\alpha - z_0)^{N(z_0)+1}\|_{\mathcal{B}} = 0.$$

Remarks

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2° Condition (D) is much easier to verify in concrete situations than the analytic Ditkin condition because we have only to deal with one given function. We call it the modified Ditkin condition. Clearly it has a local character, i.e. it depends on the degree of differentiability of functions from the algebra $\mathcal{B}$ at different points of the unit circle.

A closed ideal I of the algebra $\mathcal{B}$ is standard, according to our definition, if there exist an inner function U and a descending family of sets $H_n \subset \mathbb{T}$, $n \in \mathbb{N}_0$, such that $N(z) \geq n$ for every $z \in H_n$ and

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We obtain the following theorem.

Theorem

If $\mathcal{B}$ is a subalgebra of the disc algebra $A(\mathbb{D})$ which satisfies conditions (H1), (H2), (H3), (S), *and* (D), *then every closed ideal I of $\mathcal{B}$ with the at most countable hull*

$$h(I) = \{z \in \overline{\mathbb{D}}\colon\ f(z) = 0 \textit{ for } f \in I\}$$

is standard, i.e.

$$I = \{f \in \mathcal{B}\colon\ U|f \textit{ and } f^{(j)}(z) = 0 \text{ for } z \in h^j(I),\ j \in \mathbb{N}_0\}.$$

Now we go back to the algebra $\mathcal{B} = \mathcal{A}_1^{(1)}(\mathbb{D})$ from the Example. It is clear that it satisfies assumption (S). We have seen that $\mathcal{A}_1^{(1)}(\mathbb{D})$ does not satisfy the analytic Ditkin condition. Now we prove that it satisfies the modified Ditkin condition (D).

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First we show that this condition is satified at the point 1. We take the following sequence of functions in $\mathcal{A}_1^{(1)}(\mathbb{D})$:
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(i) The functions $\varphi_n(z) = (1-z)^{\frac{1}{n}}$ are bounded on $\overline{\mathbb{D}}$ and tend almost uniformly to 1 on $\overline{\mathbb{D}} \setminus \{1\}$. For $|z-1| < 1$ we have $|\varphi_n(z)| < 1$. Take arbitrary $\varepsilon > 0$. We may assume that $\varepsilon < 1$. Then there exists n_0 such that for all $n \geq n_0$ and $|z-1| \geq \frac{1}{2}\varepsilon$ we have $|\varphi_n(z) - 1| < \frac{1}{4}\varepsilon$. Since $|z-1| \leq 2$ for all $z \in \overline{\mathbb{D}}$ we get

$$|(z-1)(\varphi_n(z)-1)| < \varepsilon$$

for all $z \in \overline{\mathbb{D}}$ and $n \geq n_0$. This means that $(z-1)\varphi_n(z) \to z-1$ uniformly on $\overline{\mathbb{D}}$.

(ii) Since

$$(1-z)\left((z-1)\left(\varphi_n(z)-1\right)\right)' = (1-z)\left(\left(1+\frac{1}{n}\right)\varphi_n(z)-1\right)$$

similar argument as in (i) shows that
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From (i) and (ii) it follows that $\|(\alpha-1)\varphi_n-(\alpha-1)\| \to 0$ as $n \to \infty$, i.e. the modified Ditkin condition is satisfied at the point 1.

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From (i) and (ii) it follows that $\|(\alpha-1)\varphi_n-(\alpha-1)\| \to 0$ as $n\to\infty$, i.e. the modified Ditkin condition is satisfied at the point 1.

If $|z_0|=1$ and $z_0\neq 1$, then we take the functions $\psi_n(z)=(z_0-z)^{\frac{1}{n}}$, and analogously as before show that $\|(\alpha-z_0)^2\psi_n-(\alpha-z_0)^2\|\to 0$ as $n\to\infty$.

C. Agrafeuil and M. Zarrabi, *Closed ideals with countable hull in algebras of analytic functions smooth up to the boundary*, Publ. Mat. **52** (2008), 19–56.

V. M. Faĭvyševskiĭ, *The structure of the ideals of certain algebras of analytic functions*, (Russian), Dokl. Akad. Nauk SSSR **211** (1973), 537–539; translation in: Soviet Math. Dokl. **14** (1973), 1067–1070.

V. M. Faĭvyševskiĭ, *Spectral synthesis in Banach algebras of functions analytic in the disc*, (Russian), Funktsional. Anal. i Priložen. **8(3)** (1974), 85–86; translation in: Functional Anal. Appl. **8** (1974), 268–269.

J. E. Galé, P. J. Miana, and J. J. Royo, *Estimates of the Laplace transform on fractional Banach algebras*, submitted.

J. E. Galé and A. Wawrzyńczyk, *Standard ideals in weighted algebras of Korenblyum and Wiener types*, Math. Scand., in print.