

Initial-boundary Value Problems for Parabolic Equations:

1. Solve the initial-boundary value problems

$$u_t - 9u_{xx} = 0, \quad t > 0,$$

a) $0 < x < \frac{\pi}{2}, \quad u(t, 0) = u_x(t, \frac{\pi}{2}) = 0, \quad u(0, x) = x$

b) $0 < x < 1, \quad u_x(t, 0) = u(t, 1) = 0, \quad u(0, x) = 1,$

c) $0 < x < \pi, \quad u_x(t, 0) = u_x(t, \pi) = 0, \quad u(0, x) = \sin x$

d) $0 < x < 2, \quad u(t, 0) = u(t, 2) = 0, \quad u(0, x) = x^2.$

2. Solve the initial-boundary value problems

$$u_t - (u_{xx} + u_{yy}) = 0, \quad t > 0,$$

a) $0 < x < 1, \quad 0 < y < 1, \quad u(t, 0, y) = u(t, 1, y) = u_y(t, x, 0) = u(t, x, 1) = 0, \quad u(0, x, y) = -1,$

b) $0 < x < 2, \quad 0 < y < 1, \quad u(t, 0, y) = u(t, 2, y) = u(t, x, 0) = u(t, x, 1) = 0, \quad u(0, x, y) = xy,$

c) $0 < x < \pi, \quad 0 < y < \pi, \quad u(t, 0, y) = u(t, \pi, y) = u_y(t, x, 0) = u_y(t, x, \pi) = 0,$

$$u(0, x, y) = x(\pi - y).$$

3. Solve the initial-boundary value problems

$$u_t - 9u_{xx} = f(t, x), \quad t > 0,$$

a) $0 < x < \frac{\pi}{2}, \quad u(t, 0) = u_x(t, \frac{\pi}{2}) = 0, \quad u(0, x) = x, \quad f(t, x) = At$

b) $0 < x < 1, \quad u_x(t, 0) = u(t, 1) = 0, \quad u(0, x) = 1, \quad f(t, x) = Ae^{-t}$

c) $0 < x < \pi, \quad u_x(t, 0) = u_x(t, \pi) = 0, \quad u(0, x) = \sin x, \quad f(t, x) = t \sin x$