

Initial-boundary Value Problems for Hyperbolic Equations:

1. Solve the initial-boundary value problems $u_{tt} - 9u_{xx} = 0, t > 0$,
 - a) $0 < x < \frac{\pi}{2}, u(t, 0) = u_x(t, \frac{\pi}{2}) = 0, u(0, x) = x, u_t(0, x) = 1$
 - b) $0 < x < 1, u_x(t, 0) = u(t, 1) = 0, u(0, x) = 1, u_t(0, x) = x$
 - c) $0 < x < \pi, u_x(t, 0) = u_x(t, \pi) = 0, u(0, x) = \sin x, u_t(0, x) = 0$
 - d) $0 < x < 2, u(t, 0) = u(t, 2) = 0, u(0, x) = 0, u_t(0, x) = 1.$
2. Solve the initial-boundary value problems $u_{tt} - (u_{xx} + u_{yy}) = 0, t > 0$,
 - a) $0 < x < 1, 0 < y < 1, u(t, 0, y) = u(t, 1, y) = u_y(t, x, 0) = u(t, x, 1) = 0,$
 $u(0, x, y) = -1, u_t(0, x, y) = 1$
 - b) $0 < x < 2, 0 < y < 1, u(t, 0, y) = u(t, 2, y) = u(t, x, 0) = u(t, x, 1) = 0,$
 $u(0, x, y) = xy, u_t(0, x, y) = 0$
 - c) $0 < x < \pi, 0 < y < \pi, u(t, 0, y) = u(t, \pi, y) = u_y(t, x, 0) = u_y(t, x, \pi) = 0,$
 $u(0, x, y) = x(\pi - y), u_t(0, x, y) = 2.$
3. Solve the initial-boundary value problems $u_{tt} - 9u_{xx} = f(t, x), t > 0$,
 - a) $0 < x < \frac{\pi}{2}, u(t, 0) = u_x(t, \frac{\pi}{2}) = 0, u(0, x) = x, u_t(0, x) = 1, f(t, x) = At$
 - b) $0 < x < 1, u_x(t, 0) = u(t, 1) = 0, u(0, x) = 1, u_t(0, x) = x, f(t, x) = Ae^{-t}$
 - c) $0 < x < \pi, u_x(t, 0) = u_x(t, \pi) = 0, u(0, x) = \sin x, u_t(0, x) = 1, f(t, x) = t \sin x$