Na intervale [0, 1] je daná diferenciálna rovnica

$$y' = y + x^2 + 1$$
$$y(0) = 0$$

Riešte rovnicu numericky:

1. Eulerovou metódou s krokom 0.2 a 0.1 .
2. Heunovou metódou s krokom 0.2 a 0.1 . Spresnite výsledky *polovičným krokom*.
3. Klasickou metódou Runge-Kutta 4. rádu s krokom 0.2 a 0.1 .
Spresnite výsledky *polovičným krokom*.

4. Dosadením overte, že funkcia $y=3e^x-x^2-2x-3$ je riešením danej diferenciálnej rovnice. Porovnajte numericky získané výsledky s presnými hodnotami.

//

Euler

```
>> f=inline('y+x.*x+1')
```

Krok h=0.2

```
>> x=0, y=0, k=f(x,y)
x = 0,  y = 0,  k =  1

>> h=0.2; x=x+h, y=y+h*k, k=f(x,y)                    // celý riadok opakovať
x =  0.20000, y =  0.20000, k =  1.2400

x =  0.40000, y =  0.44800, k =  1.6080

x =  0.60000, y =  0.76960, k =  2.1296

x =  0.80000, y =  1.1955,  k =  2.8355

x =  1, y =  1.7626
```

Krok h=0.1

```
>> x=0, y=0, k=f(x,y)
x = 0
y = 0
k =  1
>> h=0.1; x=x+h, y=y+h*k, k=f(x,y)
x =  0.10000, y =  0.10000, k =  1.1100

x =  0.20000, y =  0.21100, k =  1.2510

x =  0.30000, y =  0.33610, k =  1.4261

x =  0.40000, y =  0.47871, k =  1.6387

x =  0.50000, y =  0.64258, k =  1.8926

x =  0.60000, y =  0.83184, k =  2.1918

x =  0.70000, y =  1.0510, k =  2.5410

x =  0.80000, y =  1.3051, k =  2.9451

x =  0.90000, y =  1.5996, k =  3.4096

x =  1.00000, y =  1.9406
```

Heun

Krok 0.2

>> x=0, y=0

x = 0, y = 0

>> h=0.2; k1=f(x,y), k2=f(x+h, y+h*k1), x=x+h, y=y+h*(k1+k2)/2

k1 = 1, k2 = 1.2400

x = 0.20000, y = 0.22400

k1 = 1.2640, k2 = 1.6368

x = 0.40000, y = 0.51408

k1 = 1.6741, k2 = 2.2089

x = 0.60000, y = 0.90238

k1 = 2.2624, k2 = 2.9949

x = 0.80000, y = 1.4281

k1 = 3.0681, k2 = 4.0417

x = 1, y = 2.1391

Krok 0.1

>> x=0, y=0
x = 0, y = 0

>> h=0.1; k1=f(x,y), k2=f(x+h, y+h*k1), x=x+h, y=y+h*(k1+k2)/2
k1 = 1, k2 = 1.1100
x = 0.10000, y = 0.10550

k1 = 1.1155, k2 = 1.2571
x = 0.20000, y = 0.22413

k1 = 1.2641, k2 = 1.4405
x = 0.30000, y = 0.35936

k1 = 1.4494, k2 = 1.6643
x = 0.40000, y = 0.51504

k1 = 1.6750, k2 = 1.9325
x = 0.50000, y = 0.69542

k1 = 1.9454, k2 = 2.2500
x = 0.60000, y = 0.90519

k1 = 2.2652, k2 = 2.6217
x = 0.70000, y = 1.1495

k1 = 2.6395, k2 = 3.0535
x = 0.80000, y = 1.4342

k1 = 3.0742, k2 = 3.5516
x = 0.90000, y = 1.7655

k1 = 3.5755, k2 = 4.1230
x = 1.00000, y = 2.1504

Spresnenie polovičným krokom v bodoch 0.2:0.2:1 –

>> yy2=[0.22400, 0.51408, 0.90238, 1.4281, 2.1391]
>> yy1=[0.22413, 0.51504, 0.90519, 1.4342, 2.1504]
>> (4*yy1-yy2)/3

ans = 0.22417 0.51536 0.90613 1.43623 2.15417

Runge-Kutta

Krok 0.2

```
>> format long
>> x=0, y=0
x = 0, y = 0

>> h=0.2; k1=f(x,y), k2=f(x+h/2, y+h/2*k1), k3=f(x+h/2, y+h/2*k2), k4=f(x+h, y+h*k3),
                                                    x=x+h, y=y+h*(k1+2*k2+2*k3+k4)/6
k1 =  1
k2 =  1.11000000000000
k3 =  1.12100000000000
k4 =  1.26420000000000
x =  0.200000000000000
y =  0.224206666666667

k1 =  1.26420666666667
k2 =  1.44062733333333
k3 =  1.45826940000000
k4 =  1.67586054666667
x =  0.400000000000000
y =  0.515468689333333

k1 =  1.67546868933333
k2 =  1.93301555826667
k3 =  1.95877024516000
k4 =  2.26722273836533
x =  0.600000000000000
y =  0.906344123818400

k1 =  2.26634412381840
k2 =  2.62297853620024
k3 =  2.65864197743842
k4 =  3.07807251930608
x =  0.800000000000000
y =  1.43659937949846

k1 =  3.07659937949846
k2 =  3.55425931744831
k3 =  3.60202531124329
k4 =  4.15700444174712
x =  1
y =  2.15480514878609
```

Krok 0.1

>> x=0, y=0
x = 0
y = 0

opakovať riadok príkazov:
>> h=0.1; k1=f(x,y), k2=f(x+h/2, y+h/2*k1), k3=f(x+h/2, y+h/2*k2), k4=f(x+h, y+h*k3), x=x+h, y=y+h*(k1+2*k2+2*k3+k4)/6

k1 = 1
k2 = 1.05250000000000
k3 = 1.05512500000000
k4 = 1.11551250000000
x = 0.100000000000000
y = 0.105512708333333

k1 = 1.11551270833333
k2 = 1.18378834375000
k3 = 1.18720212552083
k4 = 1.26423292088542
x = 0.200000000000000
y = 0.224208151129340

k1 = 1.26420815112934
k2 = 1.34991855868581
k3 = 1.35420407906363
k4 = 1.44962855903570
x = 0.300000000000000
y = 0.359576184223739

k1 = 1.44957618422374
k2 = 1.55455499343493
k3 = 1.55980393389549
k4 = 1.67555657761329
x = 0.400000000000000
y = 0.515473694498703

k1 = 1.67547369449870
k2 = 1.80174737922364
k3 = 1.80806106345989
k4 = 1.94627980084469
x = 0.500000000000000
y = 0.696163200843877

k1 = 1.94616320084388
k2 = 2.09597136088607
k3 = 2.10346176888818
k4 = 2.26650937773270
x = 0.600000000000000
y = 0.906355514812629

k1 = 2.26635551481263
k2 = 2.44217329055326
k3 = 2.45096417934029
k4 = 2.64145193274666
x = 0.700000000000000
y = 1.15125688793507

k1 = 2.64125688793507
k2 = 2.84581973233182
k3 = 2.85604787455166
k4 = 3.07686167539023
x = 0.800000000000000
y = 1.43662111755327

k1 = 3.07662111755327
k2 = 3.31295217343094
k3 = 3.32476872622482
k4 = 3.57909799017575
x = 0.900000000000000
y = 1.76880713267062

k1 = 3.57880713267062
k2 = 3.85024748930415
k3 = 3.86381950713582
k4 = 4.15518908338420
x = 1.000000000000000
y = 2.15484263615286

Spresnenie polovičným krokom v uzloch 0.2:0.2:1 –

```
>> yyr2=[0.224206666666667,  0.515468689333333,   0.906344123818400,
                1.43659937949846, 2.15480514878609]

>> yyr1=[0.224208151129340,  0.515473694498703,  0.906355514812629,
                1.43662111755327,  2.15484263615286]

>> (16*yyr1-yyr2)/15

ans =     0.224208250093518   0.515474028176394   0.906356274212244
                1.436622566756924   2.154845135310645
```