

Tabuľka Z - transformácie

Z - originál	Z - transformácia
$(a_n)_{n=0}^\infty$	$Z(a_n)_{n=0}^\infty = F(z) = \sum_{n=0}^{\infty} \frac{a_n}{z^n}$
$(a_n)_{n=0}^\infty = (0, 0, \dots, \underbrace{1}_{\text{index } m}, 0, 0, \dots) = (\delta_{mn})_{n=0}^\infty$	$Z(a_n)_{n=0}^\infty = \frac{1}{z^m}$
$(c_1 a_n + c_2 b_n)_{n=0}^\infty, \forall c_1, c_2 \in \mathbf{C}$	$Z(c_1 a_n + c_2 b_n)_{n=0}^\infty = c_1 Z(a_n)_{n=0}^\infty + c_2 Z(b_n)_{n=0}^\infty,$
$(a_n)_{n=0}^\infty = (c)_{n=0}^\infty$	$Z(c)_{n=0}^\infty = F(z) = \sum_{n=0}^{\infty} \frac{c}{z^n} = \frac{cz}{z-1}, z > 1$
$(a^n a_n)_{n=0}^\infty$	$Z(a^n a_n)_{n=0}^\infty = F\left(\frac{z}{a}\right), \forall a \neq 0$
$(na_n)_{n=0}^\infty$	$Z(na_n)_{n=0}^\infty = -zF'(z)$
Posun doprava $Z(a_n)_{n=0}^\infty = F(z) \wedge k \in \mathbf{Z}, k \geq 0, .$	
$(b_n)_{n=0}^\infty : b_n = \begin{cases} a_{n-k}, & \text{ak } n \geq k \\ 0, & \text{ak } n < k \end{cases}$	$Z(b_n)_{n=0}^\infty = \frac{1}{z^k} F(z), F(z) = Z(a_n)_{n=0}^\infty$
Posun doľava $Z(a_n)_{n=0}^\infty = F(z)$ a $k \in \mathbf{Z}, k \geq 0,$	
$(b_n)_{n=0}^\infty : b_n = a_{n+k}, n = 0, 1, \dots$	$Z(b_n)_{n=0}^\infty = z^k \left[F(z) - \sum_{n=0}^{k-1} \frac{a_n}{z^n} \right]$
$(\sum_{k=0}^n a_k)_{n=0}^\infty$	$Z(\sum_{k=0}^n a_k)_{n=0}^\infty = \frac{zF(z)}{z-1}$
Inverzná Z-transformácia:	
a) rozvoj do Laurentovho radu	
b) Podľa vety o rezíduách:	$a_n = \sum_{z_i} \text{res}_{z_i}(F(z)z^{n-1}). \text{Singularity} \in \text{Int}C$
c) priame vzorce	$a_0 = \lim_{z \rightarrow \infty} F(z), a_1 = \lim_{z \rightarrow \infty} z(F(z) - a_0),$ $a_2 = \lim_{z \rightarrow \infty} z^2 \left(F(z) - a_0 - \frac{a_1}{z} \right), \dots,$ $a_{n+1} = (-1)^{n+1} \lim_{z \rightarrow \infty} \frac{z^{n+2}}{(n+1)!} [z^n F(z)]^{(n+1)}$
d) známe obrazy	