11:00 level

**2(a)** Najdite $b \in \mathbb{R}$ aby funkcia

$$f(x,y) = \begin{cases} \dfrac{xy^3}{x^2+y^2} & \text{pre } (x,y) \neq (0,0) \\ b & \text{pre } (x,y) = (0,0) \end{cases}$$

bola spojitá v bode $\bar{a} = (0,0)$ (ak $b$ existuje)

$$\lim_{\substack{x\to 0 \\ y\to 0}} \frac{xy^3}{x^2+y^2} = \lim_{\substack{x\to 0 \\ y\to 0}} \underbrace{xy}_{\to 0} \underbrace{\frac{y^2}{x^2+y^2}}_{\leq 1} = 0$$

ohraničená

Teda $b = 0$ (3)

**2(b)** Vypočítajte $\left[\dfrac{\partial f(x,y)}{\partial y}\right]_{\substack{x=0 \\ y=0}}$ pre v 2(a) danú funkciu a nájdené $b$.

$$\left[\frac{\partial f(x,y)}{\partial y}\right]_{\substack{x=0 \\ y=0}} = \lim_{y\to 0} \frac{f(0,y) - f(0,0)}{y-0} \overset{(2)}{=}$$

$$= \lim_{y\to 0} \frac{\frac{0}{y^2} - 0}{y} \overset{(1)}{=} \lim_{y\to 0} \frac{0}{y^3} = 0 \quad (2)$$

pretože $y \neq 0$