Pomocou definície nájdite Taylorov rad funkcie f so stredom v bode a a vyšetrite jeho konvergenciu:

1. $f(z) = \sin^2 z$, $a = 0$
   $\left[\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{2n-1}}{(2n)!} z^{2n}, \text{ konverguje na } M = \mathbb{C}\right]$

2. $f(z) = \ln(iz + 2)$, $a = 1 + 2i$
   $\left[i\frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(z - 1 - 2i)^n, \text{ konverguje na } M = \{z \in \mathbb{C}; |z - 1 - 2i| < 1\}\right]$

Nájdite Taylorov rad funkcie f so stredom v bode a a vyšetrite jeho konvergenciu:

1. $f(z) = \frac{z-1}{z+1}$, $a = 0$
   $\left[-1 + 2\sum_{n=1}^{\infty} (-1)^{n+1} z^n, \text{ konverguje na } M = \{z \in \mathbb{C}; |z| < 1\}\right]$

2. $f(z) = \frac{z}{z+2}$, $a = 1$
   $\left[\frac{1}{3} + 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^{n+1}}(z - 1)^n, \text{ konverguje na } M = \{z \in \mathbb{C}; |z - 1| < 3\}\right]$

3. $f(z) = \frac{z+2}{z^2+5z+4}$, $a = 1$
   $\left[\frac{1}{3}\sum_{n=0}^{\infty} (-1)^n (2.5^{-n-1} + 2^{-n-1})(z - 1)^n, \text{ konverguje na } M = \{z \in \mathbb{C}; |z - 1| < 2\}\right]$

4. $f(z) = \frac{z}{z^2+4z+3}$, $a = 2$
   $\left[\frac{1}{2}\sum_{n=0}^{\infty} (-1)^n (3.5^{-n-1} - 3^{-n-1})(z - 2)^n, \text{ konverguje na } M = \{z \in \mathbb{C}; |z - 2| < 3\}\right]$

5. $f(z) = \frac{z+2}{z^2-2z+5}$, $a = i$
   $\left[\sum_{n=0}^{\infty} \left(-\frac{2 - 3i}{4}(1 + i)^{-n-1} - \frac{2 + 3i}{4}(1 - 3i)^{-n-1}\right)(z - i)^n, \text{ kvg. na } M = \{z \in \mathbb{C}; |z - i| < \sqrt{2}\}\right]$

6. $f(z) = \frac{z^2+i}{z^2+iz+2}$, $a = 1$
   $\left[\frac{2 + i}{5} + \frac{1}{3}\sum_{n=1}^{\infty} (-1)^n \left(\frac{1 + i}{(1 - i)^{n+1}} - \frac{1 + 4i}{(1 + 2i)^{n+1}}\right)(z - 1)^n, \text{ kvg. na } M = \{z \in \mathbb{C}; |z - 1| < \sqrt{2}\}\right]$

7. $f(z) = \mathrm{e}^{3z-2}$, $a = 1$
   $\left[\mathrm{e}\sum_{n=0}^{\infty} \frac{3^n}{n!}(z - 1)^n, \text{ konverguje na } M = \mathbb{C}\right]$

Nájdite Laurentov rad funkcie f so stredom v bode a pre medzikružie $P(a, r, R) = \{z \in \mathbb{C} : r < |z - a| < R\}$, ak:

1. $f(z) = \frac{1}{z(1-z)}$, $a = 0$, $P(0, 0, 1)$ $\qquad \left[\sum_{n=-1}^{\infty} z^n\right]$

2. $f(z) = \frac{1}{z(1-z)}$, $a = 1$, $P(1, 1, \infty)$ $\qquad \left[\sum_{n=-\infty}^{-1} (-1)^n (z - 1)^{n-1}\right]$

3. $f(z) = \frac{1}{z(z^2+1)},\ a = 0,\ P(0,0,1)$ $\qquad \left[\frac{1}{z} + \sum_{n=0}^{\infty}(-1)^{n+1}z^{2n+1}\right]$

4. $f(z) = \frac{1}{z(z^2+1)},\ a = i,\ P(i,0,1)$ $\qquad \left[\sum_{n=0}^{\infty}\frac{(-1)^n(1-2^{n+1})}{2(2i)^n}(z-i)^{n-1}\right]$

5. $f(z) = \frac{1}{z(z^2+1)},\ a = 0,\ P(0,1,\infty)$ $\qquad \left[\sum_{n=-\infty}^{-1}(-1)^{n+1}z^{2n-1}\right]$

6. $f(z) = \frac{z+1}{z(z-1)},\ a = 1,\ P(1,0,1)$ $\qquad \left[\frac{2}{z-1} + \sum_{n=0}^{\infty}(-1)^{n+1}(z-1)^n\right]$

7. $f(z) = \frac{1}{z^2+iz+2},\ a = -2i,\ P(-2i,3,\infty)$ $\qquad \left[\sum_{n=-\infty}^{-1}(3i)^{-n-1}(z+2i)^{n-1}\right]$

8. $f(z) = \frac{1}{z^2-3iz-2},\ a = 2i,\ P(2i,1,\infty)$ $\qquad \left[\sum_{n=-\infty}^{-1}(-1)^{n+1}i^{-n-1}(z-2i)^{n-1}\right]$

9. $f(z) = \frac{1}{z^2-3z+2},\ a = 1,\ P(1,1,\infty)$ $\qquad \left[\sum_{n=-\infty}^{-2}(z-1)^n\right]$

10. $f(z) = \frac{1}{z^2-3z+2},\ a = 1,\ P(1,0,1)$ $\qquad \left[(-1)\sum_{n=-1}^{\infty}(z-1)^n\right]$

11. $f(z) = \frac{1}{(z-2)^2},\ a = i,\ P(i,\sqrt{5},\infty)$ $\qquad \left[\sum_{n=0}^{\infty}\frac{(n+1)(2-i)^n}{(z-i)^{n+2}}\right]$

12. $f(z) = \frac{5z+11}{z^2+4z+3},\ a = -1,\ P(-1,0,2)$ $\qquad \left[\frac{3}{z+1} + \sum_{n=0}^{\infty}(-1)^n\frac{(z+1)^n}{2^n}\right]$

13. $f(z) = \frac{5z+11}{z^2+4z+3},\ a = -1,\ P(-1,2,\infty)$ $\qquad \left[\frac{5}{z+1} + \sum_{n=-\infty}^{-2}(-1)^{n+1}\frac{(z+1)^n}{2^n}\right]$

14. $f(z) = \frac{5z+11}{z^2+4z+3},\ a = -3,\ P(-3,0,2)$ $\qquad \left[\frac{2}{z+3} - 3\sum_{n=0}^{\infty}\frac{(z+3)^n}{2^{n+1}}\right]$

15. $f(z) = \frac{5z+11}{z^2+4z+3},\ a = -3,\ P(-3,2,\infty)$ $\qquad \left[\frac{5}{z+3} + 3\sum_{n=-\infty}^{-2}\frac{(z+3)^n}{2^{n+1}}\right]$

16. $f(z) = \frac{z}{(z^2-4)(z^2-1)},\ a = 0,\ P(0,1,2)$ $\qquad \left[-\frac{1}{3}\sum_{n=-\infty}^{-1}z^{2n+1} - \frac{1}{12}\sum_{n=0}^{\infty}4^{-n}z^{2n+1}\right]$

17. $f(z) = \frac{z^2-2z+5}{(z-2)(z^2+1)},\ a = 2,\ P(2,0,\sqrt{5})$ $\qquad \left[\frac{1}{z-2} + i\sum_{n=0}^{\infty}(-1)^n\frac{(2+i)^{n+1}-(2-i)^{n+1}}{5^{n+1}}(z-2)^n\right]$

18. $f(z) = \frac{z^2-2z+5}{(z-2)(z^2+1)},\ a = 0,\ P(0,1,2)$ $\qquad \left[2\sum_{n=-\infty}^{-1}(-1)^n z^{2n} - \sum_{n=0}^{\infty}2^{-n-1}z^n\right]$

19. $f(z) = z^2 \sin\left(\frac{\pi z+1}{z}\right)$, $a = 0$, $P(0, 0, \infty)$ $\left[-z + \sum_{n=-\infty}^{-1} \frac{(-1)^{1-n}}{(1-2n)!} z^{2n+1}\right]$

20. $f(z) = 2^z + 2^{\frac{1}{z}} - 1$, $a = 0$, $P(0, 0, \infty)$ $\left[\sum_{n=-\infty}^{-1} \frac{z^n}{(-n)!(\ln 2)^n} + \sum_{n=0}^{\infty} \frac{(\ln 2)^n z^n}{n!}\right]$

21. $f(z) = \sin\left(\frac{z}{1-z}\right)$, $a = 1$, $P(1, 0, \infty)$ $\left[-\sum_{n=0}^{\infty} \frac{\sin\left(1 + \frac{n\pi}{2}\right)}{n!(z-1)^n}\right]$