7. týždeň semestra

DÚ: Nech $f(x,y) = \frac{1}{y}\ln(x+y) + \arcsin\left(x^4 - y^2 + \frac{1}{2}\right) + \cos^2(xy^3)$. Vypočítajte $f(0,1)$, $\frac{\partial f}{\partial y}(0,1)$ a $\frac{\partial^2 f}{\partial x \partial y}(0,1)$.

Riešenie:

$$f(0,1) = \frac{1}{1}\ln(0+1) + \arcsin\left(0 - 1 + \frac{1}{2}\right) + \cos^2(0) = 0 - \frac{\pi}{6} + 1 = \mathbf{1 - \frac{\pi}{6}}$$

$$\frac{\partial f}{\partial y}(x,y) = -\frac{1}{y^2}\ln(x+y) + \frac{1}{y}\left(\frac{1}{x+y}\right) + \frac{-2y}{\sqrt{1-\left(x^4 - y^2 + \frac{1}{2}\right)^2}} - 6xy^2\cos(xy^3)\sin(xy^3)$$

$$= -\frac{\ln(x+y)}{y^2} + \frac{1}{xy+y^2} - \frac{2y}{\sqrt{1-\left(x^4 - y^2 + \frac{1}{2}\right)^2}} - 3xy^2\sin(2xy^3)$$

$$\frac{\partial f}{\partial y}(0,1) = -\frac{0}{1} + \frac{1}{0+1} - \frac{2}{\sqrt{1-\left(0-1+\frac{1}{2}\right)^2}} - 3.0.0 = -0 + 1 - \frac{2}{\sqrt{1-\left(-\frac{1}{2}\right)^2}} - 0 = 1 - \frac{2}{\sqrt{1-\frac{1}{4}}}$$

$$= 1 - \frac{2}{\sqrt{\frac{3}{4}}} = 1 - \frac{2}{\frac{\sqrt{3}}{2}} = \mathbf{1 - \frac{4}{\sqrt{3}}}$$

$$\frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}(x,y)\right) = -\frac{1}{y^2(x+y)} - \frac{y}{(xy+y^2)^2} +$$

$$\left(\frac{2y}{2}\right)\left(1-\left(x^4 - y^2 + \frac{1}{2}\right)^2\right)^{-\frac{3}{2}}\left(-2\left(x^4 - y^2 + \frac{1}{2}\right)(4x^3)\right) - 3y^2\sin(2xy^3) - 6xy^5\cos(2xy^3)$$

$$= \frac{-1}{xy^2+y^3} - \frac{y}{(xy+y^2)^2} - \frac{8x^3y\left(x^4 - y^2 + \frac{1}{2}\right)}{\left(1-\left(x^4 - y^2 + \frac{1}{2}\right)^2\right)^{\frac{3}{2}}} - 3y^2\sin(2xy^3) - 6xy^5\cos(2xy^3)$$

$$\frac{\partial^2 f}{\partial x \partial y}(0,1) = \frac{-1}{0+1} - \frac{1}{(0+1)^2} - \frac{8.0.1\left(0-1+\frac{1}{2}\right)}{\left(1-\left(0-1+\frac{1}{2}\right)^2\right)^{\frac{3}{2}}} - 3.0.0 - 6.0.1 = -1 - 1 + 0 - 0 - 0 = \mathbf{-2}$$