Vypočítajte parciálne derivácie nasledujúcich funkcií:

1. $f(x,y) = \ln\left(\mathrm{tg}\left(\frac{x}{y}\right)\right)$ $\left[\frac{\partial f}{\partial x}(x,y) = \frac{2}{y\sin\left(\frac{2x}{y}\right)},\ \frac{\partial f}{\partial y}(x,y) = \frac{-2x}{y^2\sin\left(\frac{2x}{y}\right)}\right]$

2. $f(x,y,z) = x^3y^2z + 2x - 3y + z + 5$
$\left[\frac{\partial f}{\partial x}(x,y,z) = 3x^2y^2z + 2,\ \frac{\partial f}{\partial y}(x,y,z) = 2x^3yz - 3,\ \frac{\partial f}{\partial z}(x,y,z) = x^3y^2 + 1\right]$

3. $f(x,y) = \mathrm{e}^{\sin\left(\frac{x}{y}\right)}$ $\left[\frac{\partial f}{\partial x}(x,y) = \frac{1}{y}\cos\left(\frac{x}{y}\right)\mathrm{e}^{\sin\left(\frac{x}{y}\right)},\ \frac{\partial f}{\partial y}(x,y) = -\frac{x}{y^2}\cos\left(\frac{x}{y}\right)\mathrm{e}^{\sin\left(\frac{x}{y}\right)}\right]$

4. $f(x,y) = (\ln x)^{\cos y}$ $\left[\frac{\partial f}{\partial x}(x,y) = \frac{1}{x}\cos y(\ln x)^{\cos y - 1},\ \frac{\partial f}{\partial y}(x,y) = -\sin y\ln(\ln x)(\ln x)^{\cos y}\right]$

5. $f(x,y) = x^{xy}$ $\left[\frac{\partial f}{\partial x}(x,y) = y(1+\ln x)x^{xy},\ \frac{\partial f}{\partial y}(x,y) = x^{xy+1}\ln x\right]$

6. $f(x,y,z) = zx^{\frac{1}{x}}$
$\left[\frac{\partial f}{\partial x}(x,y,z) = zx^{\frac{1}{x}-2}(1-\ln x),\ \frac{\partial f}{\partial y}(x,y,z) = 0,\ \frac{\partial f}{\partial z}(x,y,z) = x^{\frac{1}{x}}\right]$

Vypočítajte $\frac{\partial f}{\partial x}(1,\frac{\pi}{2})$, ak $f(x,y) = \ln(\sin(xy))$.
$\left[\frac{\partial f}{\partial x}(1,\frac{\pi}{2}) = 0\right]$

Vypočítajte $\frac{\partial f}{\partial x}(1,1)$, $\frac{\partial f}{\partial y}(2,1)$, ak $f(x,y) = \arcsin(\frac{x}{2}) + \sqrt{xy}$.
$\left[\frac{\partial f}{\partial x}(1,1) = \frac{2+\sqrt{3}}{2\sqrt{3}},\ \frac{\partial f}{\partial y}(2,1) = \frac{1}{\sqrt{2}}\right]$

Pomocou definície vypočítajte $\frac{\partial f}{\partial x}(0,0)$, $\frac{\partial f}{\partial y}(0,0)$, ak:

1. $$f(x,y) = \begin{cases} \dfrac{\sin(x^2+y^2)}{x^2+y^2} + x - y, & \text{pre } (x,y) \neq (0,0); \\ 1, & \text{pre } (x,y) = (0,0). \end{cases}$$
$\left[\frac{\partial f}{\partial x}(0,0) = 1,\ \frac{\partial f}{\partial y}(0,0) = -1\right]$

2. $$f(x,y) = \begin{cases} (x^2+y^2)\cos\left(\dfrac{1}{x^2+y^2}\right), & \text{pre } (x,y) \neq (0,0); \\ 0, & \text{pre } (x,y) = (0,0). \end{cases}$$
$\left[\frac{\partial f}{\partial x}(0,0) = 0,\ \frac{\partial f}{\partial y}(0,0) = 0\right]$

Vyšetrite existenciu parciálnych derivácií a diferencovatenosť funkcie f v bode $(0,0)$, ak:

1. $$f(x,y) = \begin{cases} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}, & \text{pre } (x,y) \neq (0,0); \\ 1, & \text{pre } (x,y) = (0,0). \end{cases}$$
$\left[\nexists\ \frac{\partial f}{\partial x}(0,0),\ \nexists\ \frac{\partial f}{\partial y}(0,0), \text{ funkcia } f \text{ nie je diferencovateľná v bode } (0,0)\right]$

2. $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & \text{pre } (x,y) \neq (0,0); \\ 0, & \text{pre } (x,y) = (0,0). \end{cases}$

[$\frac{\partial f}{\partial x}(0,0) = 0$, $\frac{\partial f}{\partial y}(0,0) = 0$, funkcia nie je diferencovateľná v bode $(0,0)$, pretože v ňom nie je spojitá]

3. $f(x,y) = \begin{cases} \dfrac{x^3+y^3}{x^2+y^2}, & \text{pre } (x,y) \neq (0,0); \\ 0, & \text{pre } (x,y) = (0,0). \end{cases}$

[$\frac{\partial f}{\partial x}(0,0) = 1$, $\frac{\partial f}{\partial y}(0,0) = 1$, funkcia nie je diferencovateľná v bode $(0,0)$]

4. $f(x,y) = \begin{cases} (x^2+y^2)\sin\left(\dfrac{1}{x^2+y^2}\right), & \text{pre } (x,y) \neq (0,0); \\ 0, & \text{pre } (x,y) = (0,0). \end{cases}$

[$\frac{\partial f}{\partial x}(0,0) = 0$, $\frac{\partial f}{\partial y}(0,0) = 0$, funkcia f je diferencovateľná v bode $(0,0)$]

5. $f(x,y) = \sqrt{x^2+y^2}$

[$\nexists\ \frac{\partial f}{\partial x}(0,0)$, $\nexists\ \frac{\partial f}{\partial y}(0,0)$, funkcia f nie je diferencovateľná v bode $(0,0)$]