1. $\frac{\partial f}{\partial x} = \frac{y^2(x^2+y^2) - xy^2 \cdot 2x}{(x^2+y^2)^2} = \frac{y^2(y^2-x^2)}{(x^2+y^2)^2}$ $\quad \frac{\partial f}{\partial x}(2,1) = \frac{-3}{25}$

$\frac{\partial f}{\partial y} = \frac{2xy(x^2+y^2) - xy^2 \cdot 2y}{(x^2+y^2)^2} = \frac{2x^3y}{(x^2+y^2)^2}$ $\quad \frac{\partial f}{\partial y}(2,1) = \frac{16}{25}$ $\quad f(2,1) = \frac{2}{5}$

$z - \frac{2}{5} = -\frac{3}{25}(x-2) + \frac{16}{25}(y-1)$

$df(\bar{a}, x, y) = -\frac{3}{25}(x-2) + \frac{16}{25}(y-1)$

$\left(-\frac{3}{25}, \frac{16}{25}, -1\right)$

2. $\frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot -\frac{y}{x^2} = \frac{-y}{x^2+y^2}$ $\quad \frac{\partial f}{\partial x}(1,3) = \frac{-3}{10}$

$\frac{\partial f}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} = \frac{x}{x^2+y^2}$ $\quad \frac{\partial f}{\partial y}(1,3) = \frac{1}{10}$

$\operatorname{grad} f(\bar{a}) = \left(-\frac{3}{10}, \frac{1}{10}\right)$ $\quad \frac{\partial f}{\partial \vec{e}}(\bar{a}) = \left(-\frac{3}{10}, \frac{1}{10}\right) \cdot \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = \frac{-3+\sqrt{3}}{20} < 0 \Rightarrow$ f klesá v bode $\bar{a}$ v smere $\vec{e}$

3. $\lim_{(x,y)\to(0,0)} \frac{2xy}{xy+x+y}$

$\varphi_1(t) = (t, 0)$ $\quad \lim_{t\to 0} \frac{0}{t} = 0$

$\varphi_2(t) =$ ...

$\varphi_3(t) = (t, -t)$ $\quad \lim_{t\to 0} \frac{-2t}{-t} = 2$

$\Rightarrow$ limita neexistuje.

(1) $\frac{\partial f(x,y)}{\partial x} = \frac{(y+1)(x^2+y^2) - x(y+1)\cdot 2x}{(x^2+y^2)^2} = \frac{(y+1)(y^2-x^2)}{(x^2+y^2)^2}$ $\quad \frac{\partial f}{\partial x}(1,0) = -1$

$\frac{\partial f}{\partial y} = \frac{x(x^2+y^2) - x(y+1)\cdot 2y}{(x^2+y^2)^2} = \frac{x^3+xy^2-2xy^2-2xy}{(x^2+y^2)^2} = \frac{x^3-xy^2-2xy}{(x^2+y^2)^2}$ $\quad \frac{\partial f}{\partial y}(1,0) = 1$

$f(1,0) = 1 \qquad z - 1 = -1(x-1) + 1y$

$df(\bar{a})(x,y) = -1(x-1) + 1y$

$\vec{n} = (-1, +1, -1)$

(2.) $\frac{\partial f}{\partial x} = \frac{1}{x^2+4y}\cdot 2x \qquad \frac{\partial f}{\partial x}(\bar{a}) = 2$

$\frac{\partial f}{\partial y} = \frac{1}{x^2+4y}\cdot 4 \qquad \frac{\partial f}{\partial y}(\bar{a}) = 4$

$\operatorname{grad} f(\bar{a}) = (2,4) \qquad \frac{\partial f}{\partial e}(\bar{a}) = (2,4)\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right) = 2 - \sqrt{3} > 0 \Rightarrow$ rastie v bode $\bar{a}$ v smere $\bar{e}$

(3)

limita $\quad \lim_{(x,y)\to(0,0)} \frac{x^3y - xy^3}{x^4+y^4}$

$\varphi_1(t) = (t,0) \qquad \lim_{t\to 0} \frac{0}{t^4} = 0$

$\varphi_2(t) = \ldots$ $\Rightarrow$ lim. neexistuje

$\varphi_2(t) = (t,2t) \qquad \lim_{t\to 0} \frac{2t^4 - 8t^4}{t^4+16t^4} = -\frac{6}{17}$