Týždeň 7

1. Vypočítajte derivácie funkice f, určte $D(f)$, $D(f')$

   a. $f(x) = \ln x + 2\sin x - x^3 + 2$ $\quad [\,D(f) = D(f') = (0,\infty), f' = \frac{1}{x} + 2\cos x - 3x^2\,]$

   b. $f(x) = \sqrt{x} - \sqrt[3]{x^4}$ $\quad \left[D(f) = \langle 0,\infty), D(f') = (0,\infty), f' = \frac{1}{2\sqrt{x}} - \frac{4}{3}\sqrt[3]{x}\right]$

   c. $f(x) = \operatorname{tg} x - \operatorname{arccotg} x$ $\quad \left[D(f) = \{x \in \mathbb{R} \colon x \neq (\pi/2) + k\pi\ \forall k \in \mathbb{Z}\} = D(f'), f' = \frac{1}{\cos^2 x} + \frac{1}{1+x^2}\right]$

   d. $f(x) = 2e^x - \cos x + 3^x$ $\quad [\,D(f) = D(f') = \mathbb{R}, f' = 2e^x + \sin x + 3^x \ln 3\,]$

   e. $f(x) = \frac{x^2}{2x-1}$ $\quad \left[D(f) = D(f') = \mathbb{R} \setminus \{1/2\}, f' = \frac{2x(x-1)}{(2x-1)^2}\right]$

   f. $f(x) = (x^2 - 2x + 3)e^x$, $f'(1) = ?$ $\quad [\,D(f) = d(f') = \mathbb{R}, f' = (x^2+1)e^x, f'(1) = 2e\,]$

   g. $f(x) = \frac{(x-1)\ln x}{x^2+1}$, $f'(1) = ?$ $\quad \left[D(f) = D(f') = (0,\infty), f' = \frac{x^3 - x^2 + x - 1 + (x + 2x^2 - x^3)\ln x}{x(x^2+1)^2}, f'(1) = 0\right]$

   h. $f(x) = \arcsin(2x-1)$, $f(1/2) = ?$, $f'(1/2) = ?$

      $\left[D(f) = \langle 0,1\rangle, D(f') = (0,1), f' = 1/\sqrt{x - x^2}, f(1/2) = 0, f'(1/2) = 2\right]$

   i.* $f(x) = \left(1 + \frac{1}{x}\right)^x$, $\quad \left[Df = (-\infty,-1) \cup (0,\infty), D(f') = D(f), f' = \left(1 + \frac{1}{x}\right)^x \left(\ln \frac{x+1}{x} - \frac{1}{x+1}\right)\right]$

2. Nájdite rovnicu dotyčnice ku grafu funkcie f v bode T.

   a. $f(x) = e^{1-x^2}$, $T = (-1, ?)$ $\quad [\,T = (-1,1),\ t \equiv y - 1 = 2(x+1)\,]$

   b. $f(x) = e^{1-x}\cos \pi x$, $T = (1,-1)$ $\quad [\,t \equiv y = x - 2\,]$

3. Vypočítajte $f'(a)$ pre

   a. $f(x) = |x+2|$, $a = -2$, $\quad [\,f'(-2)\ \nexists\,]$

   b. $a = 0$, $f(x) = \begin{cases} x^2 \sin \frac{1}{x}\,, & x \neq 0\,, \\ 0\,, & x = 0\,, \end{cases}$ $\quad [\,f'(0) = 0\,]$