

Goniometrické vzorce

1. $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta,$
2. $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta,$
3. $\sin 2\alpha = 2 \sin \alpha \cos \alpha, \quad \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha, \quad \cos^2 \alpha + \sin^2 \alpha = 1,$
4. $\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}, \quad \sin^2 \alpha = \frac{1 - \cos 2\alpha}{2},$
5. $\cos \alpha \cos \beta = \frac{1}{2} (\cos(\alpha + \beta) + \cos(\alpha - \beta)),$
6. $\sin \alpha \sin \beta = \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta)),$
7. $\sin \alpha \cos \beta = \frac{1}{2} (\sin(\alpha - \beta) + \sin(\alpha + \beta)).$
8. $\sin 0 = \cos \frac{\pi}{2} = \frac{\sqrt{0}}{2}, \quad \sin \frac{\pi}{6} = \cos \frac{\pi}{3} = \frac{\sqrt{1}}{2}, \quad \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2},$
9. $\sin \frac{\pi}{3} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}, \quad \sin \frac{\pi}{2} = \cos 0 = \frac{\sqrt{4}}{2},$
10. $\cos n\pi = (-1)^n, \quad \sin(2k-1)\frac{\pi}{2} = (-1)^{k+1}, \quad \cos(2k-1)\frac{\pi}{2} = 0,$

Logaritmy

1. $\log_a(xy) = \log_a x + \log_a y,$
2. $\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y,$
3. $\log_a x = \frac{\log_b x}{\log_b a} = \frac{\ln x}{\ln a},$

Derivácie

1. $(x^\alpha)' = \alpha x^{\alpha-1}.$
2. $(e^x)' = e^x.$
3. $(\sin x)' = \cos x.$
4. $(\cos x)' = -\sin x.$
5. $(\operatorname{tg} x)' = \frac{1}{\cos^2 x}.$
6. $(\operatorname{cotg} x)' = \frac{-1}{\sin^2 x}.$
7. $(\ln x)' = \frac{1}{x}.$
8. $(\log_a x)' = \frac{1}{x \ln a}.$
9. $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}.$
10. $(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}.$
11. $(\operatorname{arctg} x)' = \frac{1}{1+x^2}.$
12. $(\operatorname{arccotg} x)' = \frac{-1}{1+x^2}.$
13. $(a^x)' = a^x \ln a.$

Integrály

$$1) \quad \int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} \quad \text{pre } \alpha \neq -1.$$

$$2) \quad \int \frac{1}{x} dx = \ln|x|.$$

$$3) \quad \int \frac{f'(x)}{f(x)} dx = \ln|f(x)|.$$

$$4) \quad \int \sin x dx = -\cos x.$$

$$5) \quad \int \cos x dx = \sin x.$$

$$6) \quad \int \frac{1}{\cos^2 x} dx = \operatorname{tg} x.$$

$$7) \quad \int \frac{1}{\sin^2 x} dx = -\operatorname{cotg} x.$$

$$8) \quad \int \frac{1}{1+x^2} dx = \operatorname{arctg} x.$$

$$9) \quad \int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x.$$

$$10) \quad \int e^x dx = e^x.$$

$$11) \quad \int a^x dx = \frac{a^x}{\ln a}.$$

$$12) \quad \text{Ak } \int f(x) dx = F(x), \quad \text{potom } \int f(ax+b) dx = \frac{1}{a} F(ax+b).$$

$$13) \quad \int \frac{1}{(x-b)^2 + a^2} dx = \frac{1}{a} \operatorname{arctg} \frac{(x-b)}{a}.$$

Substitúcie

1. $\operatorname{tg} \frac{x}{2} = t, x = 2\arctg t, dx = \frac{2dt}{1+t^2}, \sin x = \frac{2t}{t^2+1}, \cos x = \frac{1-t^2}{t^2+1},$
2. $\operatorname{tg} x = t, x = \arctg t, dx = \frac{dt}{1+t^2}, \sin^2 x = \frac{t^2}{t^2+1}, \cos^2 x = \frac{1}{t^2+1}, \sin 2x = \frac{2t}{t^2+1},$
3. $\sqrt[k]{\frac{ax+b}{cx+d}} = t, \frac{ax+b}{cx+d} = t^k, x = ?, dx = ?,$
4. $a > 0, \sqrt{ax^2 + bx + c} = \sqrt{a}x \pm t, ax^2 + bx + c = ax^2 \pm 2\sqrt{axt} + t^2, x = ?, dx = ? \sqrt{ax^2 + bx + c} = ?$
5. $c > 0, \sqrt{ax^2 + bx + c} = \sqrt{c} \pm xt, ax^2 + bx + c = c \pm 2\sqrt{cxt} + x^2t^2, x = ?, dx = ? \sqrt{ax^2 + bx + c} = ?$