9 Asymptoty

Príklad 1. Nájdite definičný obor a vyšetrite asymptoty funkcie

$$f(x) = \frac{\ln(x^2-1)}{x}$$

v $\pm\infty$.

Riešenie. Funkcia f je definovaná práve keď

$$x^2 - 1 > 0 \qquad \text{(a tiež } x \neq 0\text{)}.$$

Preto

$$D_f = (-\infty, -1) \cup (1, \infty).$$

Počítajme limity

$$\begin{aligned} k &= \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\frac{\ln(x^2-1)}{x}}{x} = \lim_{x\to\infty} \frac{\ln(x^2-1)}{x^2} = \\ &= \lim_{x\to\infty} \frac{\frac{2x}{x^2-1}}{2x} = \lim_{x\to\infty} \frac{1}{x^2-1} = 0, \end{aligned}$$

$$q = \lim_{x\to\infty} f(x) - kx = \lim_{x\to\infty} \frac{\ln(x^2-1)}{x} - 0x = \lim_{x\to\infty} \frac{2x}{x^2-1} = 0.$$

Preto priamka $y = 0x + 0 = 0$ je asymptota funkcie f v $+\infty$.
Počítajme teraz limity

$$\begin{aligned} k &= \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{\frac{\ln(x^2-1)}{x}}{x} = \lim_{x\to-\infty} \frac{\ln(x^2-1)}{x^2} = \\ &= \lim_{x\to-\infty} \frac{\frac{2x}{x^2-1}}{2x} = \lim_{x\to-\infty} \frac{1}{x^2-1} = 0, \end{aligned}$$

$$q = \lim_{x\to-\infty} f(x) - kx = \lim_{x\to-\infty} \frac{\ln(x^2-1)}{x} - 0x = \lim_{x\to-\infty} \frac{2x}{x^2-1} = 0.$$

Priamka $y = 0x + 0 = 0$ je asymptota funkcie f aj v $-\infty$.