Domáce úlohy cvičenie 11 - riešenia

DÚ 1: Určte definičný obor a nájdite všetky asymptoty funkcie $f(x) = (x+3)e^{\frac{1}{x+1}}$.

Riešenie:

$$D(f) = \mathbb{R} \setminus \{-1\} = (-\infty, -1) \cup (-1, \infty)$$

ABS:

$$\lim_{x\to -1+} f(x) = \lim_{x\to -1+} (x+3)e^{\frac{1}{x+1}} = "2e^{\frac{1}{0^+}}" = "2e^{\infty}" = \infty$$

ABS funkcie f je priamka $x = -1$.

ASS v ∞:

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{(x+3)e^{\frac{1}{x+1}}}{x} = \lim_{x\to\infty} \left(\frac{x+3}{x}\right) e^{\frac{1}{x+1}} = \lim_{x\to\infty} \left(\frac{x+3}{x}\right) e^{\frac{1}{x+1}}$$
$$= \lim_{x\to\infty} \left(1 + \frac{3}{x}\right) e^{\frac{1}{x+1}} = "(1+0)e^{\frac{1}{\infty}}" = e^0 = 1$$

$$q = \lim_{x\to\infty} f(x) - kx = \lim_{x\to\infty} f(x) - x = \lim_{x\to\infty} (x+3)e^{\frac{1}{x+1}} - x = \lim_{x\to\infty} (x+3)e^{\frac{1}{x+1}} - x - 1 + 1$$
$$= \lim_{x\to\infty} (x+1)e^{\frac{1}{x+1}} - (x+1) + 2e^{\frac{1}{x+1}} + 1$$
$$= \lim_{x\to\infty} (x+1)\left(e^{\frac{1}{x+1}} - 1\right) + \lim_{x\to\infty} 2e^{\frac{1}{x+1}} + 1 = \lim_{x\to\infty} \frac{e^{\frac{1}{x+1}} - 1}{\frac{1}{x+1}} + 2 + 1$$
$$= \left| \begin{matrix} t = \dfrac{1}{x+1} \\ x \to \infty \\ t \to 0^+ \end{matrix} \right| = 3 + \lim_{t\to 0^+} \frac{e^t - 1}{t} = 3 + 1 = 4$$

ASS funkcie f v ∞ je priamka $y = kx + q = 1.x + 4 = x + 4$.

ASS v $-\infty$:

$$k = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{(x+3)e^{\frac{1}{x+1}}}{x} = \lim_{x\to-\infty} \left(\frac{x+3}{x}\right) e^{\frac{1}{x+1}} = \lim_{x\to-\infty} \left(\frac{x+3}{x}\right) e^{\frac{1}{x+1}}$$
$$= \lim_{x\to-\infty} \left(1 + \frac{3}{x}\right) e^{\frac{1}{x+1}} = "(1+0)e^{\frac{1}{-\infty}}" = e^0 = 1$$

$$q = \lim_{x\to-\infty} f(x) - kx = \lim_{x\to-\infty} f(x) - x = \lim_{x\to-\infty} (x+3)e^{\frac{1}{x+1}} - x$$
$$= \lim_{x\to-\infty} (x+3)e^{\frac{1}{x+1}} - x - 1 + 1 = \lim_{x\to-\infty} (x+1)e^{\frac{1}{x+1}} - (x+1) + 2e^{\frac{1}{x+1}} + 1$$
$$= \lim_{x\to-\infty} (x+1)\left(e^{\frac{1}{x+1}} - 1\right) + \lim_{x\to-\infty} 2e^{\frac{1}{x+1}} + 1 = \lim_{x\to-\infty} \frac{e^{\frac{1}{x+1}} - 1}{\frac{1}{x+1}} + 2 + 1$$
$$= \left| \begin{matrix} t = \dfrac{1}{x+1} \\ x \to -\infty \\ t \to 0^- \end{matrix} \right| = 3 + \lim_{t\to 0^-} \frac{e^t - 1}{t} = 3 + 1 = 4$$

ASS funkcie f v $-\infty$ je priamka $y = 1.x + 4 = x + 4$.

DÚ 2: Určte definičný obor a nájdite všetky asymptoty funkcie $f(x) = \frac{x}{\sqrt{|x^2-1|}}$.

Riešenie:

$$D(f) = \mathbb{R} \setminus \{-1,1\} = (-\infty,-1) \cup (-1,1) \cup (1,\infty)$$

ABS:

$$\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{x}{\sqrt{|x^2-1|}} = \frac{"-1"}{0^+} = -\infty$$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x}{\sqrt{|x^2-1|}} = \frac{"\ 1\ "}{0^+} = \infty$$

ABS funkcie f je priamka $x = -1$ aj priamka $x = 1$.

ASS v ∞:

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x}{\sqrt{|x^2-1|}} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2\left(1-\frac{1}{x^2}\right)}} = \lim_{x\to\infty} \frac{x}{|x|\sqrt{1-\frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{x}{x\sqrt{1-\frac{1}{x^2}}} = \lim_{x\to\infty} \frac{1}{\sqrt{1-\frac{1}{x^2}}} = \frac{1}{\sqrt{1-0}} = 1$$

ASS funkcie f v ∞ je priamka $y = 1$.

ASS v $-\infty$:

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x}{\sqrt{|x^2-1|}} = \lim_{x\to-\infty} \frac{x}{\sqrt{x^2-1}} = \lim_{x\to-\infty} \frac{x}{\sqrt{x^2\left(1-\frac{1}{x^2}\right)}} = \lim_{x\to-\infty} \frac{x}{|x|\sqrt{1-\frac{1}{x^2}}}$$

$$= \lim_{x\to-\infty} \frac{x}{-x\sqrt{1-\frac{1}{x^2}}} = \lim_{x\to-\infty} \frac{-1}{\sqrt{1-\frac{1}{x^2}}} = \frac{-1}{\sqrt{1-0}} = -1$$

ASS funkcie f v $-\infty$ je priamka $y = -1$.