11. domáca úloha – riešenie

1.

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\dfrac{(n+1)^2\,(3(n+1))!}{3^{n+1}\,((n+1)!)^3}}{\dfrac{n^2\,(3n)!}{3^n\,(n!)^3}}=$$

$$=\lim_{n\to\infty}\frac{(n+1)^2}{n^2}\cdot\frac{1}{3}\cdot\frac{(3n+3)(3n+2)(3n+1)\,(3n)!\,(n!)^3}{(3n)!\,(n+1)^3\,(n!)^3}=$$

$$=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2\cdot\frac{1}{3}\cdot\frac{(3n+3)(3n+2)(3n+1)}{(n+1)^3}=$$

$$=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2\cdot\frac{1}{3}\,\frac{n^3\left(3+\frac{3}{n}\right)\left(3+\frac{2}{n}\right)\left(3+\frac{1}{n}\right)}{n^3\left(1+\frac{1}{n}\right)^3}=$$

$$=\lim_{n\to\infty}\frac{1}{3}\cdot\frac{\left(3+\frac{3}{n}\right)\left(3+\frac{2}{n}\right)\left(3+\frac{1}{n}\right)}{\left(1+\frac{1}{n}\right)}=\frac{1}{3}\cdot 3^3=9>1,$$

teda rad ~~konverguje~~ diverguje podľa d'Alembertovho kritéria.

2.

$$\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\left(\sin\frac{3}{n^2+n+1}\right)^{\frac{n}{5}}}=$$

$$=\lim_{n\to\infty}\left(\sin\frac{3}{n^2+n+1}\right)^{\frac{1}{5}}=\left(\lim_{n\to\infty}\sin\frac{3}{n^2+n+1}\right)^{\frac{1}{5}}=$$

$$=\left[\sin\left(\lim_{n\to\infty}\frac{3}{n^2+n+1}\right)\right]^{\frac{1}{5}}=(\sin 0)^{\frac{1}{5}}=0<1,$$

teda rad konverguje podľa Cauchyho kritéria.