DÚ1 – riešenie

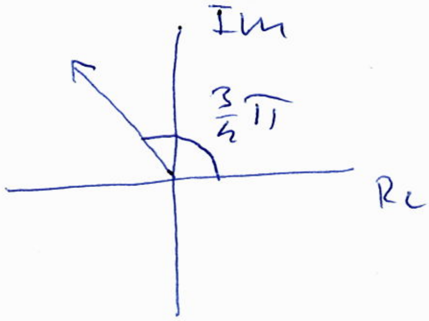
(1.) Je

$$(1+i)(1-i)=1^2-i^2=1-(-1)=1+1=2$$
$$(1+2i)(1-2i)=1^2-(2i)^2=1-(-4)=1+4=5$$
$$(1+3i)(1-3i)=1^2-(3i)^2=1-(-9)=1+9=10$$

1

Teda $z=-3i\cdot 2\cdot 5\cdot 10=-3i\cdot 100=-300i$ 1

(2.) Je $|-\sqrt{2}+\sqrt{2}i|=\sqrt{(\sqrt{2})^2+(\sqrt{2})^2}=\sqrt{2+2}=\sqrt{4}=2.$



Teda

$$-\sqrt{2}+\sqrt{2}i=2\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)=$$
$$=2\left(\cos\frac{3}{4}\pi+i\sin\frac{3}{4}\pi\right)$$

Neznámu z vyjadríme v goniometrickom tvare

$$z=r(\cos\alpha+i\sin\alpha),$$

1

teda $z^4=r^4(\cos\alpha+i\sin\alpha)^4=r^4(\cos 4\alpha+i\sin 4\alpha).$

odtiaľ plynie, že

$$r^4=2,$$
$$r=\sqrt[4]{2}$$

a

$$4\alpha=\frac{3}{4}\pi+2k\pi,\qquad k\in\mathbb{Z}.$$

Potom $\alpha=\frac{3}{16}\pi+k\frac{\pi}{2},\quad k\in\mathbb{Z},$

odkiaľ $\alpha=\frac{3}{16}\pi+k\frac{\pi}{2},\quad k=0,1,2,3$

Dostávame 4 riešenia:

$$z_0=\sqrt[4]{2}\left(\cos\frac{3}{16}\pi+i\sin\frac{3}{16}\pi\right)$$

$$z_1 = \sqrt[4]{2}\left(\cos\left(\frac{3}{16}\pi + \frac{\pi}{2}\right) + i\sin\left(\frac{3}{16}\pi + \frac{\pi}{2}\right)\right) =$$

$$= \sqrt[4]{2}\left(\cos\frac{11}{16}\pi + i\sin\frac{11}{16}\pi\right)$$

$$z_2 = \sqrt[4]{2}\left(\cos\left(\frac{3}{16}\pi + \pi\right) + i\sin\left(\frac{3}{16}\pi + \pi\right)\right) =$$ 1

$$= \sqrt[4]{2}\left(\cos\frac{19}{16}\pi + i\sin\frac{19}{16}\pi\right)$$

$$z_3 = \sqrt[4]{2}\left(\cos\left(\frac{3}{16}\pi + \frac{3}{2}\pi\right) + i\sin\left(\frac{3}{16} + \frac{3}{2}\pi\right)\right) =$$

$$= \sqrt[4]{2}\left(\cos\frac{27}{16}\pi + i\sin\frac{27}{16}\pi\right).$$

V komplexnej (Gaussovej) rovine tvoria čísla z_0, z_1, z_2, z_3 vrcholy štvorca.