$f(x) = x^4 - 18x^2 + 9$ $D(f) = \mathbb{R}$

$f'(x) = 4x^3 - 36x = 4x(x^2 - 9) = 4x(x-3)(x+3)$

$f'(x) = 0 \iff x \in \{-3, 0, 3\}$

$-3 \quad 0 \quad 3$

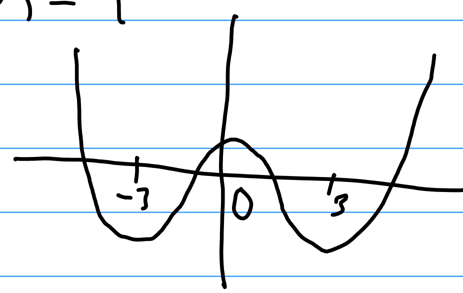
	$(-\infty, -3)$	$(-3, 0)$	$(0, 3)$	$(3, \infty)$
f'	$-$	$+$	$-$	$+$
f	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$

rast. ... na $(-3, 0)$ a na $(3, \infty)$

kles. ... na $(-\infty, -3)$ a na $(0, 3)$

lok. min. .... v $-3, 3$ $f(-3) = (-3)^4 - 18(-3)^2 + 9 = -72 = f(3)$

lok. max. .... v 0 $f(0) = 9$



$f''(x) = (4x^3 - 36x)' = 12x^2 - 36 =$

$= 12(x^2 - 3) = 12(x - \sqrt{3})(x + \sqrt{3})$

$f''(x) = 0 \iff x \in \{-\sqrt{3}, \sqrt{3}\}$

$-\sqrt{3} \quad \sqrt{3}$

	$(-\infty, -\sqrt{3})$	$(-\sqrt{3}, \sqrt{3})$	$(\sqrt{3}, \infty)$
f''	$+$	$-$	$+$
f	$\cup$	$\cap$	$\cup$

konvex. ...... na $(-\infty, -\sqrt{3})$ a na $(\sqrt{3}, \infty)$

konkáv. .... na $(-\sqrt{3}, \sqrt{3})$

inflex. body .... $-\sqrt{3}, \sqrt{3}$

$f(x) = x + \frac{1}{x}$ $\qquad D(f) = R \setminus \{0\}$

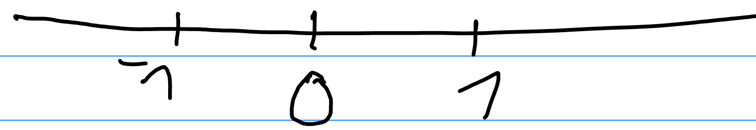
$$f'(x) = (x + x^{-1})' = 1 + (-1)x^{-2} = 1 - x^{-2} = 1 - \frac{1}{x^2}$$

$$f'(x) = 0 = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2} = \frac{(x-1)(x+1)}{x^2}$$

$\frac{1}{x^2} = 1$

$1 = x^2$

$\underline{x \in \{-1, 1\}}$



	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
f'	+	−	−	+
f	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$

rast .... na $(-\infty,-1)$ a na $(1,\infty)$

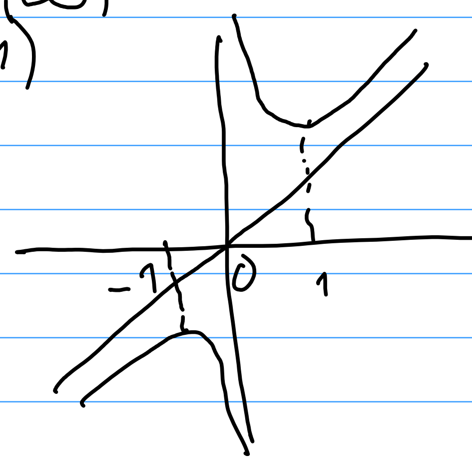
kles ...... na $(-1,0)$ a na $(0,1)$

lok. max .... v -1 $\quad f(-1) = -2$

lok. min ..... v 1 $\quad f(1) = 2$

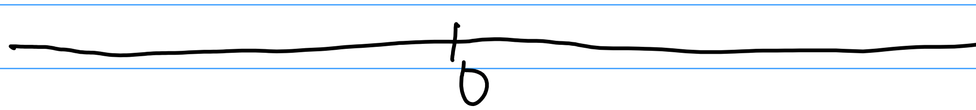
$f''(-1) = \frac{2}{(-1)^3} = -2 < 0$ $\quad$ max.

$f''(1) = \frac{2}{1^3} = 2 > 0$ $\quad$ min

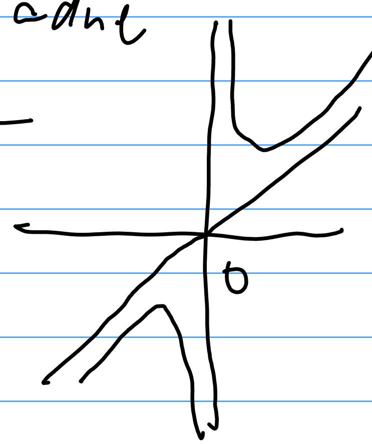


$$f''(x) = \left(1 - \frac{1}{x^2}\right)' = -(-2x^{-3}) = 2x^{-3} = \frac{2}{x^3}$$

$f''(x) = 0 \Longleftrightarrow \frac{2}{x^3} = 0 \Leftrightarrow 2 = 0$ $\quad$ žiadne



	$(-\infty,0)$	$(0,\infty)$
f''	−	+
f	$\cap$	$\cup$



konkávna na $(-\infty, 0)$
konvexná na $(0, \infty)$,
inflexné body nemá

$0?$

Taylorov polynóm

$f(x) = \frac{1}{x}$ $\quad \underline{a=2}$, st. 3

$$T_n(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$T_3(x) = f(2) + \frac{f'(2)}{1!}(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f'''(2)}{3!}(x-2)^3$$

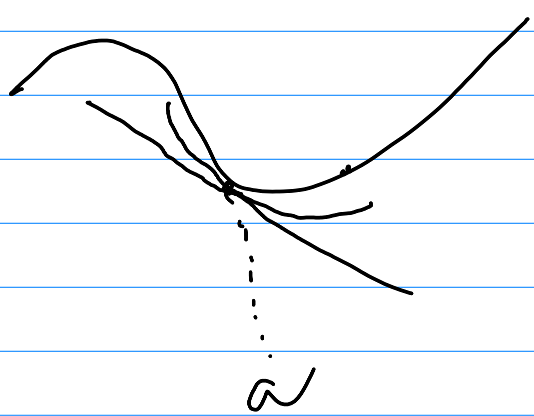
$f'(x) = -\frac{1}{x^2}$ $\qquad f'(2) = -\frac{1}{4}$

$f''(x) = \frac{2}{x^3}$ $\qquad f''(2) = \frac{1}{4}$

$f'''(x) = -\frac{6}{x^4}$ $\qquad f'''(2) = -\frac{3}{8}$

$f(2) = \frac{1}{2}$

$$T_3(x) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{4}\cdot\frac{1}{2!}(x-2)^2 - \frac{3}{8}\cdot\frac{1}{3!}(x-2)^3 =$$
$$= \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$$



$x = 2{,}1 \quad a = 2$ $\qquad f(x) = f(2{,}1) = \frac{1}{2{,}1} =$

$= \underline{0{,}4761904\dots}$

$$T_3(2{,}1) = \frac{1}{2} - \frac{1}{4}(2{,}1-2) + \frac{1}{8}(2{,}1-2)^2 - \frac{1}{16}(2{,}1-2)^3 =$$

$$= \frac{1}{2} - \frac{1}{4}0{,}1 + \frac{1}{8}0{,}01 - \frac{1}{16}0{,}001 =$$

$$= \underline{0{,}4761875}$$

$$T_1(2{,}1) = \frac{1}{2} - \frac{1}{4}(2{,}1-2) = \frac{1}{2} - \frac{1}{4}\cdot 0{,}1 = 0{,}475$$

$f(x) = e^x$ $\quad\quad$ $\underline{a=0}$ $\quad\quad$ st. 5

$$T_5(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 \checkmark$$

$f(x) = e^x$ $\quad\quad$ $f(0) = 1$

$f'(x) = e^x$ $\quad\quad$ $f'(0) = 1$

$f''(x) = e^x$ $\quad\quad$ $f''(0) = 1$

$f'''(x) = e^x$ $\quad\quad$ $f'''(0) = 1$

$f^{(4)}(x) = e^x$ $\quad\quad$ $f^{(4)}(0) = 1$

$f^{(5)}(x) = e^x$ $\quad\quad$ $f^{(5)}(0) = 1$

$$T_5(x) = 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5$$