

$$2 + 5i - (3-i)(2+3i) \quad -4-2i$$

$$(3-i)(2+3i) = 6 + 9i - 2i - 3i^2 = 6 + 7i + 3 =$$

$$= 9 + 7i$$

$$2 + 5i - (9 + 7i) = -7 - 2i$$

$$\frac{2-5i}{1+2i} = \quad -\frac{8}{5} - \frac{9}{5}i$$

$$\frac{2-5i}{1+2i} = \frac{(2-5i)(1-2i)}{(1+2i)(1-2i)} = \frac{(2-5i)(1-2i)}{1^2 - 4i^2} = \frac{(2-5i)(1-2i)}{5}$$

$$= \frac{1}{5}(2 - 4i - 5i - 10) = \frac{1}{5}(-8 - 9i) = -\frac{8}{5} - \frac{9}{5}i$$

$$\frac{a-bi}{a+bi} \quad a, b \in \mathbb{R} \quad \left[\frac{a^2 - 2abi - b^2}{a^2 + b^2} \right]$$

$$\frac{a-bi}{a+bi} = \frac{(a-bi)(a-bi)}{(a+bi)(a-bi)} = \frac{a^2 - abi - bai + b(i)^2}{a^2 - b(i)^2} =$$

$$= \frac{a^2 - 2abi - b^2}{a^2 + b^2} = \frac{a^2 - b^2}{a^2 + b^2} - \frac{2ab}{a^2 + b^2}i$$

$$x + 3y + i(2x - y) = 4 + i \quad x=1 \quad y=1$$

$$x + 3y = 4$$

$$2x - y = 1$$

$$6x - 3y = 3$$

$$7x + 0 = 7$$

$$x = 1 \quad y = 1$$

$$z = \sqrt{3} + i$$

$$z = \sqrt{3} + 1 \cdot i$$

$$\operatorname{Re} z, \operatorname{Im} z, \bar{z}, -z, |z|, \varphi$$

геометрически, экспоненциально
Гауссова норма

$$\operatorname{Re} z = \sqrt{3}$$

$$\operatorname{Im} z = 1$$

$$\bar{z} = \sqrt{3} - i$$

$$-z = -\sqrt{3} - i$$

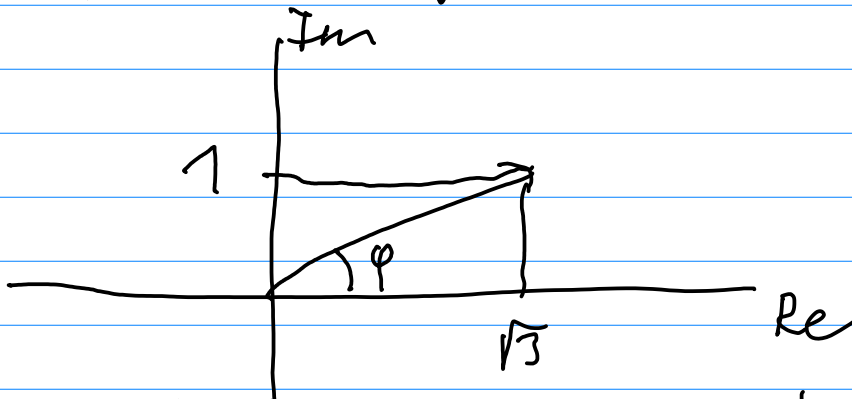
$$|z| = \sqrt{3} + 1$$

$$|z| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$$\varphi = 30^\circ = \frac{\pi}{6}$$

$$\sin \varphi = \frac{1}{|z|} = \frac{1}{2}$$

$$\Rightarrow \varphi = \frac{\pi}{6}$$

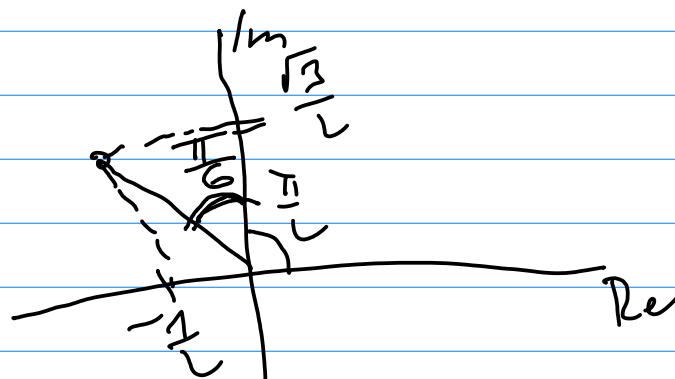
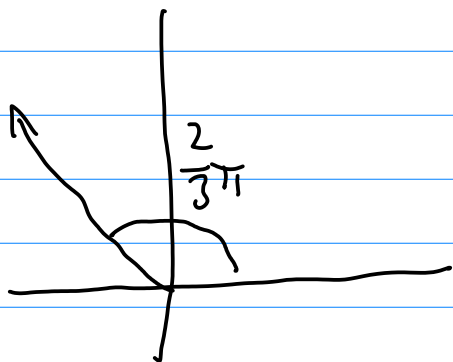


$$z = |z|(\cos \varphi + i \sin \varphi) = 2\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$$

$$z = |z|e^{i\varphi} = 2e^{i\frac{\pi}{6}}$$

$$z^3 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$\left|-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$



$$\frac{\pi}{6} + \frac{\pi}{2} = \frac{5}{6}\pi = \frac{2}{3}\pi$$

$$-\frac{1}{2} + i\frac{\sqrt{3}}{2} = 1 \cdot \left(\cos \frac{2}{3}\pi + i \sin \frac{2}{3}\pi\right)$$

$$z^3 = \cos \frac{2}{3}\pi + i \sin \frac{2}{3}\pi$$

$$z = |z|(\cos \alpha + i \sin \alpha)$$

$$z^3 = |z|^3 (\cos \alpha + i \sin \alpha)^3 = |z|^3 (\cos 3\alpha + i \sin 3\alpha)$$

$$|z|^3 (\cos 3\alpha + i \sin 3\alpha) = 1 \left(\cos \left(\frac{2}{3}\pi + 2k\pi \right) + i \sin \left(\frac{2}{3}\pi + 2k\pi \right) \right) \quad k \in \mathbb{Z}$$

$$|z|^3 = 1 \Rightarrow |z| = 1$$

$$3\alpha = \frac{2}{3}\pi + 2k\pi \quad k \in \mathbb{Z}$$

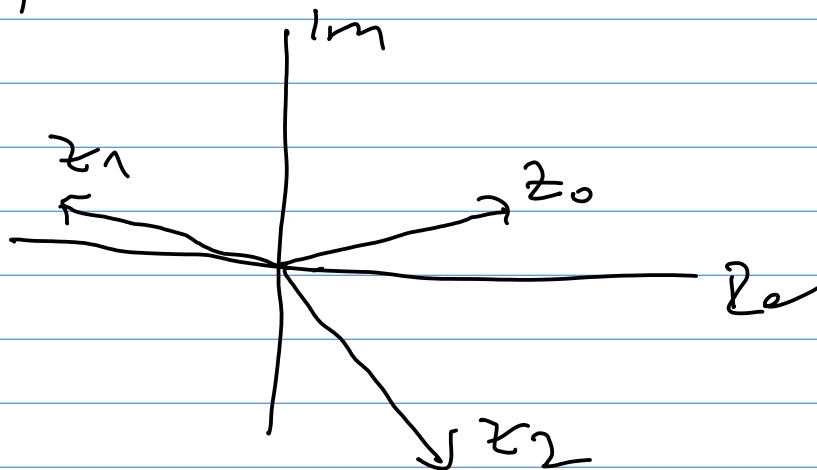
$$\alpha = \frac{2}{9}\pi + \frac{2}{3}k\pi \quad k \in \mathbb{Z}$$

$$\alpha = \frac{2}{9}\pi + \frac{2}{3}k\pi, \quad k = 0, 1, 2$$

$$z_0 = 1 \left(\cos \frac{2}{9}\pi + i \sin \frac{2}{9}\pi \right)$$

$$z_1 = 1 \left(\cos \left(\frac{2}{9}\pi + \frac{2}{3}\pi \right) + i \sin \left(\frac{2}{9}\pi + \frac{2}{3}\pi \right) \right) = \cos \frac{8}{9}\pi + i \sin \frac{8}{9}\pi$$

$$z_2 = 1 \left(\cos \left(\frac{2}{9}\pi + \frac{4}{3}\pi \right) + i \sin \left(\frac{2}{9}\pi + \frac{4}{3}\pi \right) \right) = \cos \frac{14}{9}\pi + i \sin \frac{14}{9}\pi$$



$$z = \sqrt{5} \left(\cos \frac{4}{3}\pi + i \sin \frac{4}{3}\pi \right)$$

$$w = 4 \left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5} \right)$$

$$\begin{aligned} zw &= |z||w| (\cos \alpha + i \sin \alpha) (\cos \beta + i \sin \beta) = \\ &= |z||w| (\cos(\alpha + \beta) + i \sin(\alpha + \beta)) \end{aligned}$$

$$\begin{aligned} zw &= \sqrt{5} \cdot 4 \left(\cos \left(\frac{4}{3}\pi + \frac{1}{5}\pi \right) + i \sin \left(\frac{4}{3}\pi + \frac{1}{5}\pi \right) \right) = \\ &= \sqrt{5} \cdot 4 \left(\cos \frac{23}{15}\pi + i \sin \frac{23}{15}\pi \right) \end{aligned}$$

$$\begin{aligned} w^3 &= \left[4 \left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5} \right) \right]^3 = 4^3 \left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5} \right)^3 = \\ &= 4^3 \left(\cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5} \right) \end{aligned}$$

$$\frac{z}{w} = \frac{|z|}{|w|} (\cos(\alpha - \beta) + i \sin(\alpha - \beta)) =$$

$$= \frac{\sqrt{5}}{4} \left(\cos \left(\frac{4}{3}\pi - \frac{\pi}{5} \right) + i \sin \left(\frac{4}{3}\pi - \frac{\pi}{5} \right) \right) =$$

$$= \frac{\sqrt{5}}{4} \left(\cos \frac{17}{15}\pi + i \sin \frac{17}{15}\pi \right)$$

