$$\lim_{x\to\infty} \frac{3x^2+2x+1}{5x^2-1} = \qquad\qquad \frac{3}{5}$$

$$= \lim_{x\to\infty} \frac{x^2\left(3+\frac{2}{x}+\frac{1}{x^2}\right)}{x^2\left(5-\frac{1}{x^2}\right)} = \lim_{x\to\infty} \frac{3+\underbrace{\frac{2}{x}}_{\to 0}+\underbrace{\frac{1}{x^2}}_{\to 0}}{5-\underbrace{\frac{1}{x^2}}_{\to 0}} =$$

$$= \frac{3+0+0}{5-0} = \frac{3}{5}$$

$$\lim_{x\to\infty} \frac{2x^3+3x+2}{x^2-x+1} = \lim_{x\to\infty} \frac{x^2\left(2x+\frac{3}{x}+\frac{2}{x^2}\right)}{x^2\left(1-\frac{1}{x}+\frac{1}{x^2}\right)} = \qquad\qquad \begin{matrix}\infty \\ 0\end{matrix}$$

$$= \lim_{x\to\infty} \frac{\underbrace{2x}_{\to\infty}+\underbrace{\frac{2}{x}}_{\to 0}+\underbrace{\frac{3}{x^2}}_{\to 0}}{1-\underbrace{\frac{1}{x}}_{\to 0}+\underbrace{\frac{1}{x^2}}_{\to 0}} = \infty$$

$$\lim_{x\to\infty} \frac{x^3\left(2+\frac{3}{x^2}+\frac{2}{x^3}\right)}{x^2\left(1-\frac{1}{x}+\frac{1}{x^2}\right)} = \lim_{x\to\infty} \underbrace{x}_{\to\infty} \cdot \frac{2+\underbrace{\frac{3}{x^2}}_{\to 0}+\underbrace{\frac{2}{x^3}}_{\to 0}}{1-\underbrace{\frac{1}{x}}_{\to 0}+\underbrace{\frac{1}{x^2}}_{\to 0}} = \infty$$

$$\lim_{x\to\infty} \frac{5x^2-4x-2}{6x^3+2x^2-x} = \lim_{x\to\infty} \frac{x^2\left(5-\frac{4}{x}-\frac{2}{x^2}\right)}{x^3\left(6+\frac{2}{x}-\frac{1}{x^2}\right)} = \qquad\qquad 0$$

$$= \lim_{x\to\infty} \underbrace{\frac{1}{x}}_{\to 0} \cdot \underbrace{\frac{5-\frac{4}{x}-\frac{2}{x^2}}{6+\frac{2}{x}-\frac{1}{x^2}}}_{\to \frac{5}{6}} = 0$$

$$\lim_{x\to-\infty} \frac{x^3-3x+2}{2x^2+4} = \lim_{x\to-\infty} \frac{x^3\left(1-\frac{3}{x^2}+\frac{2}{x^3}\right)}{x^2\left(2+\frac{4}{x^2}\right)} = \qquad\qquad -\infty$$

$$= \lim_{x\to\infty} x \cdot \underbrace{\frac{1-\frac{3}{x^2}+\frac{2}{x^3}}{2+\frac{4}{x^2}}}_{\to \frac{1}{2}} = -\infty$$

$$\lim_{x\to\infty}\frac{\sqrt{4x^4+2x^2+3}}{5x^2-3x+4}=\lim_{x\to\infty}\frac{\sqrt{x^4\left(4+\frac{2}{x^2}+\frac{3}{x^4}\right)}}{x^2\left(5-\frac{3}{x}+\frac{4}{x^2}\right)}= \qquad \frac{2}{5}$$

$$=\lim_{x\to\infty}\frac{x^2\sqrt{4+\frac{2}{x^2}+\frac{3}{x^4}}}{x^2\left(5-\frac{3}{x}+\frac{4}{x^2}\right)}=\lim_{x\to\infty}\frac{\sqrt{4+\frac{2}{x^2}+\frac{3}{x^4}}}{5-\frac{3}{x}+\frac{4}{x^2}}=\frac{\sqrt{4}}{5}=\frac{2}{5}$$

$$\lim_{x\to\infty}\sqrt{x^2+1}-x= \qquad \underline{\infty-\infty}$$

$$=\lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right)\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}=\lim_{x\to\infty}\frac{x^2+1-x^2}{\sqrt{x^2+1}+x}=$$

$$=\lim_{x\to\infty}\frac{1}{\underbrace{\sqrt{x^2+1}}_{\downarrow\,\infty}+\underbrace{x}_{\downarrow\,\infty}}=0$$

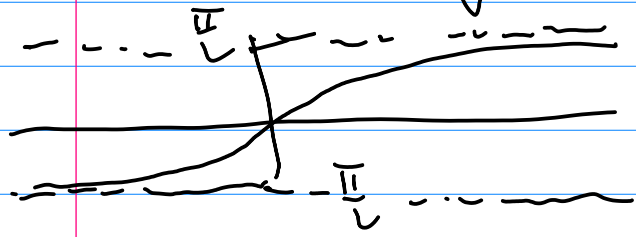
$$\lim_{x\to\infty}\sqrt{x^2+x}-x= \qquad \underline{\infty-\infty} \quad 7$$

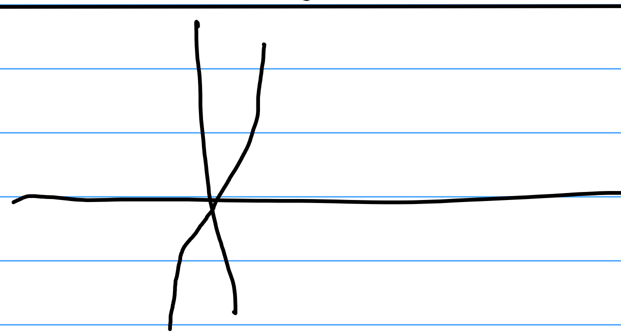
$$=\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)\frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}=$$

$$=\lim_{x\to\infty}\frac{x}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{x}{\sqrt{x^2\left(1+\frac{1}{x}\right)}+x}=$$

$$=\lim_{x\to\infty}\frac{x}{x\sqrt{1+\frac{1}{x}}+x}=\lim_{x\to\infty}\frac{1}{\sqrt{1+\underbrace{\frac{1}{x}}_{\to 0}}+1}=\frac{1}{\sqrt{1}+1}=\frac{1}{2}$$

$$f(x)=\operatorname{arctg}\frac{3}{x^2}$$





$D(f) = \mathbb{R} \setminus \{0\}$

$f(0) = \lim_{x \to 0} f(x) = \lim_{x \to 0} \operatorname{arccotg} \frac{3}{x^2} = \lim_{t \to \infty} \operatorname{arccotg} t = \frac{\pi}{2}$

f je spoj. v a

$\lim_{x \to a} f(x) = f(a)$

$f(0) = \frac{\pi}{2}$

$$f(x) = \begin{cases} \frac{\sin 2x}{3x} & x < 0 \\ a & x = 0 \\ \frac{\sqrt{4x+1} - 1}{3x} & x > 0 \end{cases}$$

$$\lim_{x \to 0} f(x) = f(0) = a$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin 2x}{3x} = \frac{2}{3} \qquad \frac{2}{3}$$

$$= \frac{2}{3}$$

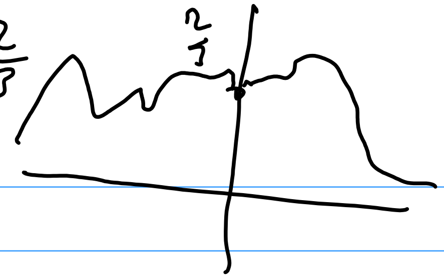
$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\sqrt{4x+1} - 1}{3x} =$$

$$= \lim_{x \to 0^+} \frac{\sqrt{4x+1} - 1}{3x} \cdot \frac{\sqrt{4x+1} + 1}{\sqrt{4x+1} + 1} = \lim_{x \to 0^+} \frac{4x+1-1}{3x(\sqrt{4x+1}+1)} =$$

$$= \lim_{x \to 0^+} \frac{4x}{3x(\sqrt{4x+1}+1)} = \lim_{x \to 0^+} \frac{4}{3(\sqrt{4x+1}+1)} = \frac{4}{3(\sqrt{1}+1)} =$$

$$= \frac{2}{3}$$

$\lim_{x \to 0^-} f(x) = \frac{2}{3}$ $\quad$ $\lim_{x \to 0^+} f(x) = \frac{2}{3}$



$$\lim_{x \to 0} f(x) = \frac{2}{3}$$

$$a = f(0) = \lim_{x \to 0} f(x) = \frac{2}{3}$$