$$\lim_{x\to\infty}\frac{3x^2+2x+1}{5x^2-1}=\lim_{x\to\infty}\frac{x^2\left(3+\frac{2}{x}+\frac{1}{x^2}\right)}{x^2\left(5-\frac{1}{x^2}\right)}=$$

$\frac{3}{5}$

$$=\lim_{x\to\infty}\frac{3+\overset{\to 0}{\frac{2}{x}}+\overset{\to 0}{\frac{1}{x^2}}}{5-\underset{\to 0}{\frac{1}{x^2}}}=\frac{3+0+0}{5-0}=\frac{3}{5}$$

$$\lim_{x\to\infty}\frac{2x^3+3x+2}{x^2-x+1}=\lim_{x\to\infty}\frac{x^3\left(2+\frac{3}{x^2}+\frac{2}{x^3}\right)}{x^2\left(1-\frac{1}{x}+\frac{1}{x^2}\right)}=$$

∞

$$=\lim_{x\to\infty}\underset{\downarrow\ \infty}{x}\cdot\underset{\to\ 2>0}{\frac{2+\frac{3}{x^2}+\frac{2}{x^3}}{1-\frac{1}{x}+\frac{1}{x^2}}}=\infty$$

$$\lim_{x\to\infty}\frac{5x^2-4x-2}{6x^3+2x^2-x}=\lim_{x\to\infty}\frac{x^2\left(5-\frac{4}{x}-\frac{2}{x^2}\right)}{x^3\left(6+\frac{2}{x}-\frac{1}{x^2}\right)}=$$

0

$$=\lim_{x\to\infty}\underset{\downarrow\ 0}{\frac{1}{x}}\cdot\underset{\to\ \frac{5}{6}}{\frac{5-\frac{4}{x}-\frac{2}{x^2}}{6+\frac{2}{x}-\frac{1}{x^2}}}=0\cdot\frac{5}{6}=0$$

$$\lim_{x\to-\infty}\frac{x^3-3x+2}{2x^2+4}=\lim_{x\to-\infty}\frac{x^3\left(1-\frac{3}{x^2}+\frac{2}{x^3}\right)}{x^2\left(2+\frac{4}{x^2}\right)}=$$

$-\infty$

$$=\lim_{x\to-\infty}x\cdot\underset{\to\ \frac{1}{2}>0}{\frac{1-\overset{\to 0}{\frac{3}{x^2}}+\overset{\to 0}{\frac{2}{x^3}}}{2+\underset{\to 0}{\frac{4}{x^2}}}}=-\infty$$

$$\lim_{x\to\infty} \frac{\sqrt{4x^4+2x^3+3}}{5x^2-3x+4} = \lim_{x\to\infty} \frac{\sqrt{x^4\left(4+\frac{2}{x}+\frac{3}{x^4}\right)}}{x^2\left(5-\frac{3}{x}+\frac{4}{x^2}\right)} =$$

$$\frac{2}{5}$$

$$= \lim_{x\to\infty} \frac{x^2\sqrt{4+\frac{2}{x}+\frac{3}{x^4}}}{x^2\left(5-\frac{3}{x}+\frac{4}{x^2}\right)} = \lim_{x\to\infty} \frac{\sqrt{4+\frac{2}{x}+\frac{3}{x^4}}}{5-\frac{3}{x}+\frac{4}{x^2}} = \frac{\sqrt{4+0+0}}{5-0+0} = \frac{2}{5}$$

$$\lim_{x\to\infty} \sqrt{x^2+1} - x = \lim_{x\to\infty} x\sqrt{1+\frac{1}{x^2}} - x$$

1

0

$$= \lim_{x\to\infty} \left(\sqrt{x^2+1}-x\right)\cdot\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x} = \lim_{x\to\infty} \frac{x^2+1-x^2}{\sqrt{x^2+1}+x} =$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{x^2+1}+x} = 0$$

$\infty - \infty \quad 0$

$\infty - \infty$

$$\lim_{x\to\infty} \sqrt{x^2+x} - x =$$

$\infty - \infty$

$1 \quad \frac{1}{2}$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2+x}-x\right)\left(\sqrt{x^2+x}+x\right)}{\sqrt{x^2+x}+x} =$$

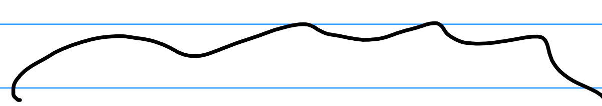
$$= \lim_{x\to\infty} \frac{x^2+x-x^2}{\sqrt{x^2+x}+x} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+x}+x} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2\left(1+\frac{1}{x}\right)}+x} =$$

$$= \lim_{x\to\infty} \frac{x}{x\sqrt{1+\frac{1}{x}}+x} = \lim_{x\to\infty} \frac{x}{x\left(\sqrt{1+\frac{1}{x}}+1\right)} = \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x}}+1} =$$

$$= \frac{1}{\sqrt{1}+1} = \underline{\underline{\frac{1}{2}}}$$

$$\lim_{x\to\infty} \underset{\infty}{(x+a)} - \underset{\infty}{x} = a$$

$$f(a) = \lim_{x\to a} f(x)$$



$$f(x) = \frac{1-3^x}{9^x-1}$$

$a=1$

$a=0$

A N

$D_f = \mathbb{R} \setminus \{0\}$

$$f(x) = \frac{1-3^x}{(3^2)^x-1} = \frac{1-3^x}{(3^x)^2-1} = \frac{1-3^x}{}$$

$$\lim_{x\to 0}\frac{1-3^x}{9^x-1} = \lim_{x\to 0}\frac{1-3^x}{3^{2x}-1} = \lim_{x\to 0}\frac{1-3^x}{(3^x)^2-1^2} =$$

$$= \lim_{x\to 0}\frac{1-3^x}{(3^x-1)(3^x+1)} = \lim_{x\to 0}\frac{-1}{3^x+1} = -\frac{1}{2}$$

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0}\frac{1-3^x}{9^x-1} = \lim_{x\to 0}\frac{1-3^x}{3^{2x}-1} = \lim_{x\to 0}\frac{1-3^x}{(3^x)^2-1^2} =$$

$$= \lim_{x\to 0}\frac{1-3^x}{(3^x-1)(3^x+1)} = \lim_{x\to 0}\frac{-1}{3^x+1} = \frac{-1}{3^0+1} = -\frac{1}{2}$$

$$\lim_{x\to 0} f(x) = -\frac{1}{2} \qquad f(0) = -\frac{1}{2}$$

$$f(0) = \lim_{x\to 0} f(x)$$

$$f(x) = \begin{cases} \frac{\sin 2x}{3x} & x<0 \\ a & x=0 \\ \frac{\sqrt{4x+1}-1}{3x} & x>0 \end{cases}$$

1

$$\lim_{x\to 0} f(x) = f(0) = a = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin 2x}{3x} = \frac{2}{3}$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{4x+1}-1}{3x} = \lim_{x\to 0^+} \frac{\sqrt{4x+1}-1}{3x} \cdot \frac{\sqrt{4x+1}+1}{\sqrt{4x+1}+1} =$$

$$= \lim_{x\to 0^+} \frac{4x+1-1}{(\sqrt{4x+1}+1)3x} = \lim_{x\to 0^+} \frac{4x}{3x(\sqrt{4x+1}+1)} =$$

$$= \lim_{x\to 0^+} \frac{4}{3(\sqrt{4x+1}+1)} = \frac{4}{3(\sqrt{1}+1)} = \frac{4}{6} = \frac{2}{3}$$

$$a = \frac{2}{3}$$

$$a = f(0) = \lim_{x\to 0} f(x)$$

$$\lim_{x\to 0} f(x) = \boxed{\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \frac{2}{3}}$$