$$\lim_{x\to 1}\frac{x+5}{x^3+1}=\frac{1+5}{1+1}=3$$

$$\lim_{x\to 1}\frac{x^2-3x+2}{x^2-1}=\lim_{x\to 1}\frac{(x-1)(x-2)}{\underline{(x-1)(x+1)}}=\lim_{x\to 1}\frac{x-2}{\underline{x+1}}=$$

$$=\frac{1-2}{1+1}=-\frac{1}{2}$$

$$\lim_{x\to 0}\frac{2x}{\sqrt{1+3x}-1}=\lim_{x\to 0}\frac{2x}{\sqrt{1+3x}}-\lim_{x\to 0}\frac{2x}{1}$$

$$\frac{a}{c+d}=\frac{a}{c}+\frac{a}{d}$$

$$\lim_{x\to 0}\frac{2x}{\sqrt{1+3x}-1}=\lim_{x\to 0}\frac{2x}{\sqrt{1+3x}-1}\cdot\frac{\sqrt{1+3x}+1}{\sqrt{1+3x}+1}=$$

$$=\lim_{x\to 0}\frac{2x\left(\sqrt{1+3x}+1\right)}{1+3x-1}=\lim_{x\to 0}\frac{2x\left(\sqrt{1+3x}+1\right)}{3x}=$$

$$=\lim_{x\to 0}\frac{2\left(\sqrt{1+3x}+1\right)}{3}=\frac{2\left(\sqrt{1}+1\right)}{3}=\frac{4}{3}$$

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{\sqrt{x^3}-1}=$$

$$a^3-b^3=(a-b)\underline{\left(a^2+ab+b^2\right)}$$

$$=\lim_{x\to 1}\frac{\sqrt{x}-1}{\left(\sqrt{x}\right)^3-1^3}=\lim_{x\to 1}\frac{\sqrt{x}-1}{\left(\sqrt{x}-1\right)\left(\left(\sqrt{x}\right)^2+\sqrt{x}\cdot 1+1^2\right)}=$$

$$= \lim_{x\to 1} \frac{\sqrt{x}-1}{(\sqrt{x}-1)\left((\sqrt{x})^2+\sqrt{x}+1\right)} =$$

$$\sqrt{x^3} = (x^3)^{\frac{1}{2}} = x^{\frac{3}{2}}$$

$$(\sqrt{x})^3 = \dots$$

$$= \lim_{x\to 1} \frac{1}{(\sqrt{x})^2+\sqrt{x}+1} = \frac{1}{1+1+1} = \frac{1}{3}$$

$$\lim_{x\to 0} \frac{\sin 2x}{7x} = \lim_{x\to 0} \frac{2}{7} \lim_{x\to 0} \frac{\sin 2x}{2x} =$$

$$\frac{2}{7}$$

$$= \frac{2}{7} \cdot 1 = \frac{2}{7}$$

$$\frac{2}{7} \frac{\sin 2x}{2x} = \frac{\sin 2x}{7x}$$

$$\lim_{x\to 0} \frac{\sin(2x)}{2x} = 1$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{\sin 2x}{2x} =$$

$$f(x) = \frac{\sin x}{x}$$

$$g(y) = 2y$$

$$t = 2x$$

$$f(g(y)) = \frac{\sin 2y}{2y}$$

$$= \lim_{t\to 0} \frac{\sin t}{t} = 1$$

$$\lim_{x\to 0} \frac{1}{2x \cot 3x} =$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$= \lim_{x\to 0} \frac{1}{2x \frac{\cos 3x}{\sin 3x}} = \lim_{x\to 0} \frac{\sin 3x}{2x \cos 3x} =$$

$$= \lim_{x\to 0} \frac{\sin 3x}{2x} \cdot \frac{1}{\cos 3x} = \lim_{x\to 0} \frac{\sin 3x}{2x} \cdot \lim_{x\to 0} \frac{1}{\cos 3x} =$$

$= \frac{3}{2} \cdot 1 = \frac{3}{2}$

$\frac{\sin 3x}{2x} = \frac{3}{2} \frac{\sin 3x}{3x} \to 1$

$$\lim_{x \to 0} \frac{\sin 2x}{\sqrt{2x+4} - 2} =$$

4

$$= \lim_{x \to 0} \frac{\sin 2x}{\sqrt{2x+4} - 2} \cdot \frac{\sqrt{2x+4} + 2}{\sqrt{2x+4} + 2} =$$

$$= \lim_{x \to 0} \frac{\sin 2x \left(\sqrt{2x+4} + 2\right)}{2x + 4 - 4} =$$

$$= \lim_{x \to 0} \frac{\sin 2x \left(\sqrt{2x+4} + 2\right)}{2x} =$$

$$= \lim_{x \to 0} \frac{\sin 2x}{2x} \cdot \lim_{x \to 0} \left(\sqrt{2x+4} + 2\right) = 1 \cdot \left(\sqrt{4} + 2\right) = 4$$

$$\lim_{x \to 1} \frac{\sin (x^2 - 1)}{x - 1} =$$

$\sin 2$

$\boxed{2}$

$$\lim_{x \to 1} \frac{\sin \left((x-1)(x+1)\right)}{x - 1} = \lim_{x \to 1} \sin (x+1)$$

$\frac{\sin (xy)}{x} = \sin y$

$$\lim_{x \to 1} \frac{\sin (x^2 - 1)}{x - 1} = \lim_{x \to 1} \sin \left((x-1)(x+1)\right)$$

$$= \lim_{x \to 1} \frac{\sin (x^2 - 1) \, (x+1)}{(x-1)(x+1)} = \lim_{x \to 1} \frac{\sin (x^2 - 1)}{x^2 - 1} \cdot (x+1) =$$

$$= \lim_{x \to 1} \frac{\sin (x^2 - 1)}{x^2 - 1} \cdot \lim_{x \to 1} (x+1) = \lim_{t \to 0} \frac{\sin t}{t} \cdot 2 =$$

$t = x^2 - 1 \qquad x \to 1 \qquad t \to 0$

$= 1 \cdot 2 = 2$

$$\lim_{x \to 0} x^2 \sin \frac{1}{x^2} = 0$$

$x^2 \to 0$ ohraničená

0

$$\lim_{x \to \infty} \frac{\sin^2 x}{2^x} = \lim_{x \to \infty} \sin^2 x \cdot \frac{1}{2^x} = 0$$

$\sin^2 x$ ohranič. $\qquad \frac{1}{2^x} \to 0$

0