$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

$$AB = \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix} \quad \underline{\begin{pmatrix} 4 & 11 \\ 10 & 25 \end{pmatrix}} \qquad \begin{pmatrix} 4 & 11 \\ 10 & 25 \end{pmatrix}$$

$$BA = \begin{pmatrix} 11 & 16 \\ 13 & 18 \end{pmatrix} \qquad \begin{pmatrix} 11 & 16 \\ 13 & 18 \end{pmatrix}$$

$$AB \stackrel{?}{=} BA$$

$$A,\ A^T,\ (A^T)^T$$

$$A = \begin{pmatrix} 1 & 5 \\ 2 & 6 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & 2 \\ 5 & 6 \end{pmatrix} \qquad (A^T)^T = \begin{pmatrix} 1 & 5 \\ 2 & 6 \end{pmatrix} = A$$

$$(A^T)^T \stackrel{?}{=} A$$

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \qquad A^{-1} \qquad \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right) \sim R_2 - R_1 \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{array}\right) \sim$$

$$\sim \tfrac{1}{2} \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right) \sim R_1 + R_2 \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad AA^{-1} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & +\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^{-1}A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \qquad \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \sim R_2 - R_1 \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \sim$$

$$\sim_{R_2 \leftrightarrow R_3} \left(\begin{array}{ccc|ccc} 1&1&0&1&0&0\\ 0&1&1&0&0&1\\ 0&-1&1&-1&1&0\end{array}\right) \sim_{R_3+R_2} \left(\begin{array}{ccc|ccc} 1&1&0&1&0&0\\ 0&1&1&0&0&1\\ 0&0&2&-1&1&1\end{array}\right) \sim$$

$$\sim_{\frac{1}{2}R_3} \left(\begin{array}{ccc|ccc} 1&1&0&1&0&0\\ 0&1&1&0&0&1\\ 0&0&1&-\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\end{array}\right) \sim_{R_2-R_3} \left(\begin{array}{ccc|ccc} 1&1&0&1&0&0\\ 0&1&0&\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\ 0&0&1&-\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\end{array}\right) \sim$$

$$\sim_{R_1-R_2} = \left(\begin{array}{ccc|ccc} 1&0&0&\frac{1}{2}&\frac{1}{2}&-\frac{1}{2}\\ 0&1&0&\frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\ 0&0&1&-\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\end{array}\right) \qquad A^{-1} = \begin{pmatrix} \frac{1}{2}&\frac{1}{2}&-\frac{1}{2}\\ \frac{1}{2}&-\frac{1}{2}&\frac{1}{2}\\ -\frac{1}{2}&\frac{1}{2}&\frac{1}{2}\end{pmatrix}$$

$$AA^{-1} = \dots = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \qquad A^{-1}A = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

$$A = \begin{pmatrix}1&2\\3&4\end{pmatrix} \qquad \det A = |A| = -2$$

$$\det A = 1\cdot 4 - 2\cdot 3 = 4 - 6 = -2$$

$$A = \begin{pmatrix}2&3&1\\3&2&1\\1&1&2\end{pmatrix} \qquad \det A \qquad -8$$

$$\det A = \left|\begin{matrix}2&3&1\\3&2&1\\1&1&2\end{matrix}\right| \begin{matrix}2&3\\3&2\\1&1\end{matrix} = 8+3+3-(2+2+18) =$$

$$= 14 - 22 = -8$$

$$A = \begin{pmatrix}1&3&4&2\\2&3&4&1\\1&2&3&4\\3&4&1&2\end{pmatrix} \qquad \det A = (-1)^{1+1}\cdot 1 \left|\begin{matrix}3&4&1\\2&3&4\\4&1&2\end{matrix}\right| +$$

$$+(-1)^{1+2}\, 3 \left|\begin{matrix}2&4&1\\1&3&4\\3&1&2\end{matrix}\right| + (-1)^{1+3}\cdot 4 \left|\begin{matrix}2&3&1\\1&2&4\\3&4&2\end{matrix}\right| + (-1)^{1+4}\, 2 \left|\begin{matrix}2&3&4\\1&2&3\\3&4&1\end{matrix}\right|$$

$$A = \begin{pmatrix}1&2&-1&1\\2&1&3&1\\0&0&2&2\\1&3&2&-1\end{pmatrix}$$

$$\det A = (-1)^{3+1}\, 0 \begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & 1 \\ 3 & 2 & -1 \end{vmatrix} + (-1)^{3+2}\, 0 \begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & -1 \end{vmatrix} +$$

$$+ (-1)^{3+3}\, 2 \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 3 & -1 \end{vmatrix} + (-1)^{3+4}\, 2 \begin{vmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{vmatrix} =$$

$$= (-1)^{3+1}\, 2 \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 3 & -1 \end{vmatrix} + (-1)^{3+4}\, 2 \begin{vmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{vmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & -9 \end{pmatrix} \qquad \det A = 1 \cdot (-1) - (9) = 9$$

$$A = \begin{pmatrix} 5 & -3 \\ -2 & -1 \end{pmatrix} \qquad A^{-1} = \frac{1}{\det A} \operatorname{adj} A$$

$$\operatorname{adj} A = (\tilde{a}_{ij})^T \qquad \tilde{a}_{ij} = (-1)^{i+j} \det A_{ij}$$

$$\det A = 5 \cdot (-1) - (-2)(-3) = 5 - 6 = -11$$

$$(\tilde{a}_{ij}) = \begin{pmatrix} (-1)^{1+1}(-1) & (-1)^{1+2}(-2) \\ (-1)^{2+1}(-3) & (-1)^{2+2}\, 5 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 3 & 5 \end{pmatrix}$$

$$(\tilde{a}_{ij})^T = \begin{pmatrix} -1 & 3 \\ 2 & 5 \end{pmatrix} \qquad A^{-1} = -\frac{1}{11} \begin{pmatrix} -1 & 3 \\ 2 & 5 \end{pmatrix}$$

$$x_1 + 2x_2 - 3x_3 = -5$$
$$3x_1 + 4x_2 + 5x_3 = 10$$
$$2x_1 + 5x_2 - 7x_3 = -9$$

$$d = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 4 & 5 \\ 2 & 5 & -7 \end{vmatrix} = 4$$

$$d_1 = \begin{vmatrix} -5 & 2 & -3 \\ 10 & -4 & 5 \\ 9 & 5 & -7 \end{vmatrix} = 7 \qquad x_1 = \frac{d_1}{d} = \frac{7}{4}$$

$$d_2 = \begin{vmatrix} 1 & -5 & -3 \\ 3 & 10 & 5 \\ 2 & -9 & -4 \end{vmatrix} = 39 \qquad x_2 = \frac{d_2}{d} = \frac{39}{4}$$

$$d_3 = \begin{vmatrix} 1 & 2 & -5 \\ 3 & -4 & 10 \\ 2 & 5 & -9 \end{vmatrix} = 35 \qquad x_3 = \frac{d_3}{d} = \frac{35}{4}$$

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