

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \quad AB \quad BA$$

$$AB = \begin{pmatrix} 4 & 11 \\ 10 & 25 \end{pmatrix}$$

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$$BA = \begin{pmatrix} 11 & 16 \\ 13 & 18 \end{pmatrix}$$

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$$AB \neq BA$$

$$AB = BA$$

$$AB \neq BA$$

$$A, A^T, (A^T)^T, \dots$$

$$((A^T)^T)^T$$

$$A = \begin{pmatrix} 5 & 6 \\ 2 & 4 \end{pmatrix} \quad A^T = \begin{pmatrix} 5 & 2 \\ 6 & 4 \end{pmatrix}$$

$$(A^T)^T = \begin{pmatrix} 5 & 6 \\ 2 & 4 \end{pmatrix} = A$$

матрица A

матрица A^T

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$A^{-1}$$

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \checkmark$$

$$\left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array} \right) \sim R_2 - R_1 \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \end{array} \right) \sim \frac{1}{2} R_2 \left(\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array} \right)$$

$$\sim R_1 + R_2 \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array} \right)$$

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$AA^{-1} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad A^{-1}A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad A^{-1}$$

$$\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{3}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array} \right) \sim$$

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\sim R_2 - R_1 \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array} \right) \sim R_2 \leftrightarrow R_3 \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 1 & 0 \end{array} \right) \sim$$

$$\sim R_3 + R_2 \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & -1 & 1 & 1 \end{array} \right) \sim \frac{1}{2} R_3 \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array} \right) \sim$$

$$\sim R_2 - R_3 \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array} \right) \sim R_1 - R_2 \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array} \right)$$

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad \det A = |A| =$$

$$\begin{matrix} 4 \\ -2 \end{matrix}$$

$$= 1 \cdot 4 - 2 \cdot 3 = 4 - 6 = -2$$

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

$$\det A$$

$$-8$$

$$\begin{vmatrix} 2 & 3 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} \begin{vmatrix} 2 & 3 \\ 3 & 2 \\ 1 & 1 \end{vmatrix} = 8 + 3 + 3 - (2 + 2 + 18) = 14 - 22 = -8$$

$$\begin{vmatrix} 2 & 3 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$A = \begin{pmatrix} 1 & 3 & 4 & 2 \\ 2 & 3 & 4 & 1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} \quad \det A$$

$$\det A = (-1)^{1+1} \cdot 1 \cdot \begin{vmatrix} 3 & 4 & 1 \\ 2 & 3 & 4 \\ 4 & 1 & 2 \end{vmatrix} + (-1)^{1+2} \cdot 3 \cdot \begin{vmatrix} 2 & 4 & 1 \\ 1 & 3 & 4 \\ 3 & 1 & 2 \end{vmatrix} +$$

$$+ (-1)^{1+3} \cdot 4 \cdot \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 4 \\ 3 & 4 & 2 \end{vmatrix} + (-1)^{1+4} \cdot 2 \cdot \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 3 & 4 & 1 \end{vmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 2 & 1 & 3 & 1 \\ 0 & 0 & 2 & 2 \\ 1 & 2 & 3 & -1 \end{pmatrix} \quad \det A$$

$$\det A = (-1)^{3+1} \cdot 0 \cdot \begin{vmatrix} 2 & -1 & 1 \\ 1 & 3 & 1 \\ 2 & 3 & -1 \end{vmatrix} + (-1)^{3+2} \cdot 0 \cdot \begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 1 \\ 1 & 3 & -1 \end{vmatrix} +$$

$$+ (-1)^{3+3} \cdot 2 \cdot \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & -1 \end{vmatrix} + (-1)^{3+4} \cdot 2 \cdot \begin{vmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 1 & 2 & 3 \end{vmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & -9 \end{pmatrix}$$

9

$$\det A = 1(-1)(-9) = 9$$

$$A = \begin{pmatrix} 5 & -3 \\ -2 & -1 \end{pmatrix} \quad A^{-1}$$

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A \quad \operatorname{adj} A = (\tilde{a}_{ij})^T$$

$$\tilde{a}_{ij} = (-1)^{i+j} \det A_{ij}$$

$$(\tilde{a}_{ij}) = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} \\ \tilde{a}_{21} & \tilde{a}_{22} \end{pmatrix} = \begin{pmatrix} (-1)^{1+1}(-1) & (-1)^{1+2}(-2) \\ (-1)^{2+1}(-3) & (-1)^{2+2}5 \end{pmatrix} =$$

$$\begin{pmatrix} -1 & 2 \\ 3 & 5 \end{pmatrix} \quad \text{adj } A = (\tilde{a}_{ij})^T = \begin{pmatrix} -1 & 3 \\ 2 & 5 \end{pmatrix}$$

$$\det A = -11$$

$$A^{-1} = -\frac{1}{11} \begin{pmatrix} -1 & 3 \\ 2 & 5 \end{pmatrix}$$

$$x_1 + 2x_2 - 3x_3 = -5$$

$$3x_1 - 4x_2 + 5x_3 = 10$$

$$2x_1 + 5x_2 + 7x_3 = -9$$

$$d = \begin{vmatrix} 1 & 2 & -3 \\ 3 & -4 & 5 \\ 2 & 5 & 7 \end{vmatrix} = 4$$

$$d_1 = \begin{vmatrix} -5 & 2 & -3 \\ 10 & -4 & 5 \\ -9 & 5 & 7 \end{vmatrix} = 7 \quad x_1 = \frac{d_1}{d} = \frac{7}{4}$$

$$d_2 = \begin{vmatrix} 1 & -5 & -3 \\ 3 & 10 & 5 \\ 2 & -9 & 7 \end{vmatrix} = 39 \quad x_2 = \frac{d_2}{d} = \frac{39}{4}$$

$$d_3 = \begin{vmatrix} 1 & 2 & -5 \\ 3 & -4 & 10 \\ 2 & 5 & -9 \end{vmatrix} = 35 \quad x_3 = \frac{d_3}{d} = \frac{35}{4}$$

