$$\begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 3 & 1 & -2 & 1 \\ 1 & 3 & -2 & 5 \end{array}\right)$$

$$\begin{aligned} x_1 + x_2 - x_3 &= 2 \\ 3x_1 + x_2 - 2x_3 &= 1 \\ x_1 + 3x_2 - 2x_3 &= 5 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 3 & 1 & -2 & 1 \\ 1 & 3 & -2 & 5 \end{array}\right) \sim \begin{matrix} R_2 - 3R_1 \\ R_3 - R_1 \end{matrix} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -2 & 1 & -5 \\ 0 & 2 & -1 & 3 \end{array}\right) \sim$$

$$\sim \begin{matrix} \\ R_3 + R_2 \end{matrix} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -2 & 1 & -5 \\ 0 & 0 & 0 & -2 \end{array}\right)$$

$$0x_1 + 0x_2 + 0x_3 = -2$$

$$0 = -2$$

$P = \emptyset$

$$\left(\begin{array}{ccc|c} 2 & 3 & -5 & 2 \\ 1 & 2 & -3 & -1 \\ 3 & 4 & -5 & 5 \end{array}\right) \sim$$

$$2x_3 = 8$$

$$x_3 = 4$$

$$x_1 = x_2 = 0$$

$$x_1 = 7 \quad x_2 = -4 \quad x_3 = 0$$

$$\sim \begin{matrix} R_1 \leftrightarrow R_2 \\ \\ \end{matrix} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 2 & 3 & -5 & 2 \\ 3 & 4 & -5 & 5 \end{array}\right) \sim \begin{matrix} R_2 - 2R_1 \\ R_3 - 3R_1 \\ \\ \end{matrix} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -1 & 1 & 4 \\ 0 & -2 & 4 & 8 \end{array}\right) \sim$$

$$\sim \begin{matrix} (-1)R_2 \\ \\ \\ \end{matrix} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & 1 & -1 & -4 \\ 0 & -2 & 4 & 8 \end{array}\right) \sim \begin{matrix} \\ R_3 + 2R_2 \\ \\ \end{matrix} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 2 & 0 \end{array}\right) \sim$$

$$\sim \left(\tfrac{1}{2}\right)R_3 \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 0 \end{array}\right) \sim \begin{matrix} R_2+3R_3 \\ R_1+R_3 \end{matrix} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \end{array}\right) \sim$$

$$\sim R_1-2R_2 \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$x_1=7 \quad x_2=-4 \quad x_3=0$

$P=\{(7,-4,0)\}$

$$x_1+2x_2-3x_3=1$$
$$2x_1+3x_2-5x_3=-1$$
$$x_2-x_1=3$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 2 & 3 & -5 & -1 \\ 0 & 1 & -1 & 3 \end{array}\right) \sim$$

$x_3=a \quad x_2=a+3$

$x_1=a-5$

$a\in\mathbb{R}$

$$\sim R_2-2R_1 \left(\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & -1 & 1 & -3 \\ 0 & 1 & -1 & 3 \end{array}\right) \sim R_3+R_2 \left(\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & -1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim$$

$$\sim (-1)R_2 \left(\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim R_1-2R_2 \left(\begin{array}{ccc|c} 1 & 0 & -1 & -5 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_3=a \quad x_2=a+3 \quad x_1=a-5 \qquad a\in\mathbb{R}$

$$\left(\begin{array}{cccc|c} 2 & 0 & 0 & 3 & 18 \\ 2 & 1 & -1 & 3 & 4 \\ 3 & 2 & -2 & 3 & -1 \end{array}\right) \sim \left(\tfrac{1}{2}\right)R_1 \left(\begin{array}{cccc|c} 1 & 0 & 0 & \frac{3}{2} & 9 \\ 2 & 1 & -1 & 3 & 4 \\ 3 & 2 & -2 & 3 & -1 \end{array}\right) \sim$$

$$\sim \begin{matrix} R_2-2R_1 \\ R_3-3R_1 \end{matrix} \left(\begin{array}{cccc|c} 1 & 0 & 0 & \frac{3}{2} & 9 \\ 0 & 1 & -1 & 0 & -14 \\ 0 & 2 & -2 & -\frac{3}{2} & -28 \end{array}\right) \sim \left(\tfrac{1}{2}\right)R_3 \left(\begin{array}{cccc|c} 1 & 0 & 0 & \frac{3}{2} & 9 \\ 0 & 1 & -1 & 0 & -14 \\ 0 & 1 & -1 & -\frac{3}{4} & -14 \end{array}\right)_2$$

$$\sim \overset{R_3 - R_2}{\begin{pmatrix} 1 & 0 & 0 & \frac{3}{2} \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -\frac{3}{4} \end{pmatrix}} \left| \begin{matrix} 9 \\ -14 \\ 0 \end{matrix} \right. \sim \overset{(-\frac{4}{3})R_3}{\begin{pmatrix} 1 & 0 & 0 & \frac{3}{2} \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}} \left| \begin{matrix} 9 \\ -14 \\ 0 \end{matrix} \right. \sim$$

$$\sim \overset{R_1 - \frac{3}{2}R_3}{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}} \left| \begin{matrix} 9 \\ -14 \\ 0 \end{matrix} \right.$$

$x_1 = 9, \; x_4 = 0,$

$x_2 - x_3 = -14$

$x_2 = x_3 - 14$ $\quad x_2 = a - 14$

$x_3 = a$ $\quad a \in \mathbb{R}$

$$P = \{(9, a-14, a, 0) \; ; \; a \in \mathbb{R}\}$$