

$$\begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 3 & 1 & -2 & 1 \\ 1 & 3 & -2 & 5 \end{array} \right) \sim$$

$$x_1 + x_2 - x_3 = 2$$

$$3x_1 + x_2 - 2x_3 = 1$$

$$x_1 + 3x_2 - 2x_3 = 5$$

$$x_1 = 0,5 \quad x_2 = 2,5 \quad x_3 = 0$$

$$\sim \begin{array}{l} R_2 - 3R_1 \\ R_3 - R_1 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -2 & 1 & -5 \\ 0 & 2 & -1 & 3 \end{array} \right) \sim$$

$$\sim R_3 + R_2 \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -2 & 1 & -5 \\ 0 & 0 & 0 & -2 \end{array} \right) \quad \begin{array}{l} 0x_1 + 0x_2 + 0x_3 = -2 \\ 0 = -2 \end{array}$$

$$\left(\begin{array}{ccc|c} 2 & 3 & -5 & 2 \\ 1 & 2 & -3 & -1 \\ 3 & 4 & -5 & 5 \end{array} \right) \sim$$

НЕМА' РЕШ.

$$x_1 = 7, \quad x_2 = -4, \quad x_3 = 0$$

$$\sim R_1 \leftrightarrow R_2 \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 2 & 3 & -5 & 2 \\ 3 & 4 & -5 & 5 \end{array} \right) \sim \begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & -1 & 1 & 4 \\ 0 & -2 & 4 & 8 \end{array} \right) \sim$$

$$\sim (-1)R_2 \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & 1 & -1 & -4 \\ 0 & -2 & 4 & 8 \end{array} \right) \sim R_3 + 2R_2 \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 2 & 0 \end{array} \right) \sim$$

$$\sim \frac{1}{2}R_3 \left(\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 0 \end{array} \right) \sim \begin{array}{l} R_1 + 3R_3 \\ R_2 + R_3 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \end{array} \right) \sim$$

$$\sim R_1 - 2R_2 \left(\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \end{array} \right) \quad \begin{array}{l} x_1 = 7, x_2 = -4, x_3 = 0 \\ P = \{(7, -4, 0)\} \end{array}$$

$$\begin{array}{l} x_1 + 2x_2 - 3x_3 = 1 \\ 2x_1 + 3x_2 - 5x_3 = -1 \\ x_2 - x_3 = 3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 2 & 3 & -5 & -1 \\ 0 & 1 & -1 & 3 \end{array} \right) \sim$$

$$-5+a$$

$$x_1 = 1+a \quad x_2 = 3+a \quad x_3 = a$$

$$\cancel{x_1 = -5 \quad x_2 = 3 \quad x_3 = 0}$$

$$\sim R_2 - 2R_1 \left(\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & -1 & 1 & -3 \\ 0 & 1 & -1 & 3 \end{array} \right) \sim \begin{array}{l} R_3 + R_2 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & -1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array} \right) \sim$$

$$\sim (-1)R_2 \left(\begin{array}{ccc|c} 1 & 2 & -3 & 1 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right) \sim R_1 - 2R_2 \left(\begin{array}{ccc|c} 1 & 0 & -1 & -5 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$x_1 = -5+a, x_2 = 3+a, x_3 = a$$

$$P = \{(-5+a, 3+a, a); a \in \mathbb{R}\}$$

$$x_3 = a, x_2 = 3+a, x_1 = -5+a$$

$$\left(\begin{array}{cccc|c} 2 & 0 & 0 & 3 & 18 \\ 2 & 1 & -1 & 3 & 4 \\ 3 & 2 & -2 & 3 & -1 \end{array} \right) \sim \frac{1}{2}R_1 \left(\begin{array}{cccc|c} 1 & 0 & 0 & \frac{3}{2} & 9 \\ 2 & 1 & -1 & 3 & 4 \\ 3 & 2 & -2 & 3 & -1 \end{array} \right) \sim$$

$$\begin{array}{l} \sim R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \left(\begin{array}{cccc|c} 1 & 0 & 0 & \frac{3}{2} & 9 \\ 0 & 1 & -1 & 0 & -14 \\ 0 & 2 & -2 & -\frac{3}{2} & -28 \end{array} \right) \sim R_3 - 2R_2 \left(\begin{array}{cccc|c} 1 & 0 & 0 & \frac{3}{2} & 9 \\ 0 & 1 & -1 & 0 & -14 \\ 0 & 0 & 0 & -\frac{3}{2} & 0 \end{array} \right) \sim$$

$$\sim \left(-\frac{2}{3} \right) R_3 \left(\begin{array}{cccc|c} 1 & 0 & 0 & \frac{3}{2} & 9 \\ 0 & 1 & -1 & 0 & -14 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right) \sim R_1 - \frac{3}{2} R_3 \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 9 \\ 0 & 1 & -1 & 0 & -14 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right)$$

$$x_4 = 0, x_3 = a, x_2 = -14 + a, x_1 = 9$$

$$P = \{(9, -14 + a, a, 0) ; a \in \mathbb{R}\}$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 4 & -3 \\ 2 & -1 & -1 & 5 & 3 \\ 2 & 1 & -3 & 3 & 9 \end{array} \right) \sim R_2 - 2R_1 \quad R_3 - 2R_1 \left(\begin{array}{cccc|c} 1 & -2 & 1 & 4 & -3 \\ 0 & 3 & -3 & -3 & 9 \\ 0 & 5 & -5 & -5 & 15 \end{array} \right)$$

$$\sim \frac{1}{3} R_2 \quad \frac{1}{5} R_3 \left(\begin{array}{cccc|c} 1 & -2 & 1 & 4 & -3 \\ 0 & 1 & -1 & -1 & 3 \\ 0 & 1 & -1 & -1 & 3 \end{array} \right) \sim R_3 - R_2 \left(\begin{array}{cccc|c} 1 & -2 & 1 & 4 & -3 \\ 0 & 1 & -1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\sim R_1 + 2R_2 \left(\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 3 \\ 0 & 1 & -1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \quad \begin{array}{l} x_4 = a \quad x_3 = b \\ x_2 = 3 + a + b \\ x_1 = 3 - 2a + b \end{array}$$

$$P = \{(3 - 2a + b, 3 + a + b, b, a) ; a, b \in \mathbb{R}\}$$

$$x_1 - x_3 + 2x_4 = 3$$

$$x_1 - b + 2a = 3$$

$$x_1 = 3 + b - 2a$$





