$f(x) = x + \frac{1}{x-1}$ $\qquad$ $y = x$

$x = 1$ $\qquad$ $D(f) = R \setminus \{1\}$

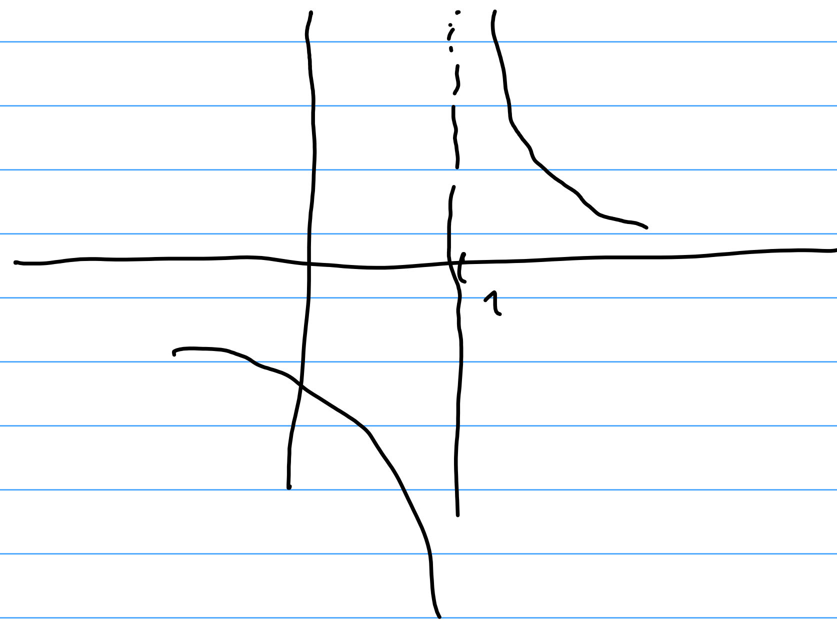
$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-}\left(x + \frac{1}{x-1}\right) = 1 + \lim\limits_{x\to 1^-} \frac{1}{x-1} =$

$= -\infty$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+}\left(x + \frac{1}{x-1}\right) = 1 + \lim\limits_{x\to 1^+} \frac{1}{x-1} =$

$= \infty$

as. bez smer. $\qquad$ $x = 1$



as. so smer.

$f(x) = x + \frac{1}{x-1}$ $\qquad$ $\underline{y = x}$

$k = \lim\limits_{x\to\infty} \frac{f(x)}{x} = \lim\limits_{x\to\infty}\left(1 + \frac{1}{x(x-1)}\right) = 1 + 0 = 1$

$q = \lim\limits_{x\to\infty}(f(x) - 1.x) = \lim\limits_{x\to\infty}\left(x + \frac{1}{x-1} - 1.x\right) =$

$$= \lim_{x \to \infty} \frac{1}{x-1} = 0$$

v ∞

$$y = kx + q$$
$$y = 1 \cdot x + 0$$
$$y = x$$

v $-\infty$

$$k = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \left(1 + \frac{1}{x(x-1)}\right) = 1 + 0 = 1$$

$$q = \lim_{x \to -\infty} \left(f(x) - 1 \cdot x\right) = \lim_{x \to -\infty} \left(x + \frac{1}{x-1} - 1 \cdot x\right) =$$

$$= \lim_{x \to -\infty} \frac{1}{x-1} = 0$$

$$y = kx + q$$

v $-\infty$ $\quad y = 1 \cdot x + 0$

$$y = x$$

$y = x$ v $\pm\infty$

$$f(x) = \frac{x^3}{2(x+1)^2}$$

1. Def. obor, body nespojitosti
2. nulové body
3. intervaly monotónnosti, lok. extrémy
4. intervaly konvexnosti, konkávnosti, inflexné body
5. asymptoty
6. graf

(1.) $D(f) = \mathbb{R} \setminus \{-1\}$

body nespojitosti

(2.) nulové body

-1

$f(x) = 0$

$x = 0$

$f(-1) = 0$?

$$\frac{x^3}{2(x+1)^2} = 0$$

$$x^3 = 0$$

$$x = 0$$

(3.) $f'(x) = \left[\frac{x^3}{2(x+1)^2}\right]' = \frac{x^2(x+3)}{2(x+1)^3}$

$$f'(x) = \frac{3x^2 \cdot 2(x+1)^2 - x^3 4(x+1)}{4(x+1)^4} =$$

$$= \frac{6x^2(x+1) - 4x^3}{4(x+1)^3} = \frac{2x^3 + 6x^2}{4(x+1)^3} =$$

$$= \frac{x^3 + 3x^2}{2(x+1)^3} = \frac{x^2(x+3)}{2(x+1)^3}$$

stac. body $0, -3$

$$\frac{x^2(x+3)}{2(x+1)^3} = 0$$

$$x^2(x+3) = 0$$

$$x = 0, \quad x = -3$$

4.

	$(-\infty,-3)$	$(-3,-1)$	$(-1,0)$	$(0,\infty)$
f'	+	−	+	+
f	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$

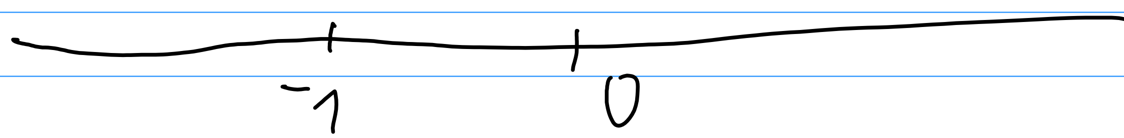
$$f'(x) = \frac{x^2(x+3)}{2(x+1)^3}$$

rastie na $(-\infty,-3)$, $(-1,0)$, $(0,\infty)$

klesá na $(-3,-1)$

lok. max. v -3

$$f''(x) = \left[\frac{x^2(x+3)}{2(x+1)^3}\right]' = \frac{3x}{(x+1)^4}$$



$$f''(x) = 0 \iff \frac{3x}{(x+1)^4} = 0 \iff 3x = 0 \iff x = 0$$

	$(-\infty,-1)$	$(-1,0)$	$(0,\infty)$
f''	−	−	+
f	$\cap$	$\cap$	$\cup$

infl. bod 0

5. $D(f) = \mathbb{R} \setminus \{-1\}$ ∞

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} \frac{x^3}{2(x+1)^2} = -\infty$$

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} \frac{x^3}{2(x+1)^2} = -\infty$$

as. bez smer. $x = -1$

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\frac{x^3}{2(x+1)^2}}{x} = \lim_{x\to\infty} \frac{x^3}{2x(x+1)^2} =$$

$$= \lim_{x\to\infty} \frac{x^3 \cdot 1}{x^3 \cdot 2\left(1+\frac{1}{x}\right)^2} = \lim_{x\to\infty} \frac{1}{2\left(1+\frac{1}{x}\right)^2} = \frac{1}{2}$$

$$q = \lim_{x\to\infty} (f(x) - kx) = \lim_{x\to\infty} \left[\frac{x^3}{2(x+1)^2} - \frac{1}{2}x\right] = -1$$

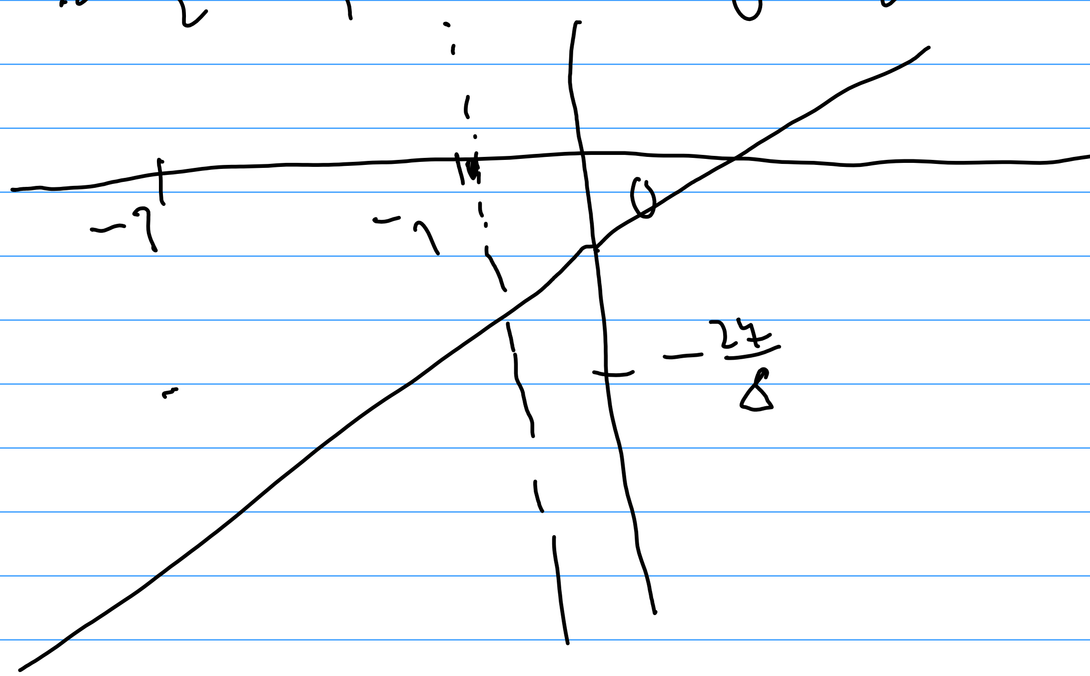
as. so smer. v ∞

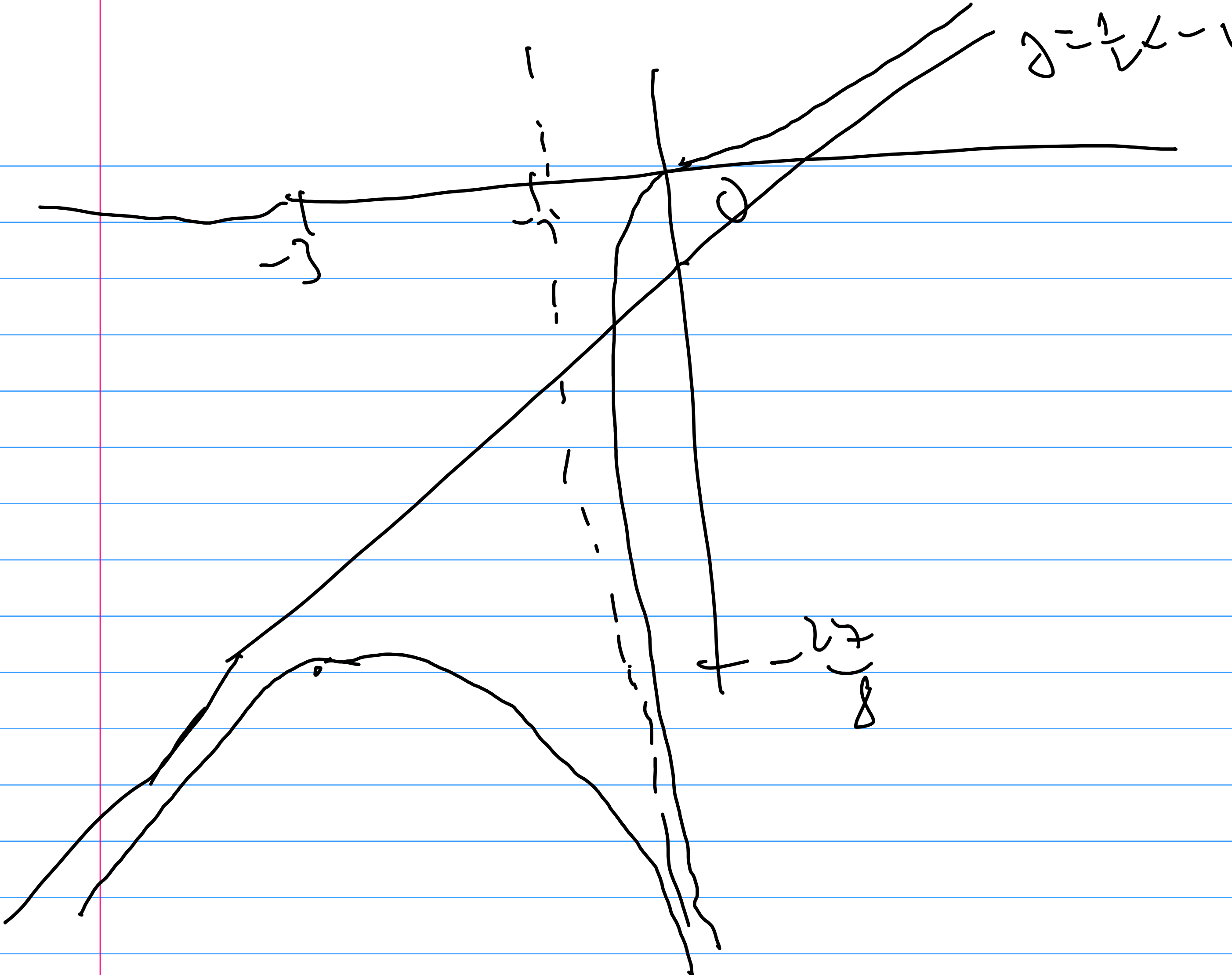
$$y = \frac{1}{2}x - 1$$

as. so smer. v $-\infty$

$k = \frac{1}{2}$ $q = -1$ $y = \frac{1}{2}x - 1$

6.





y = 1/2 x − 1
-3
0
-27/8