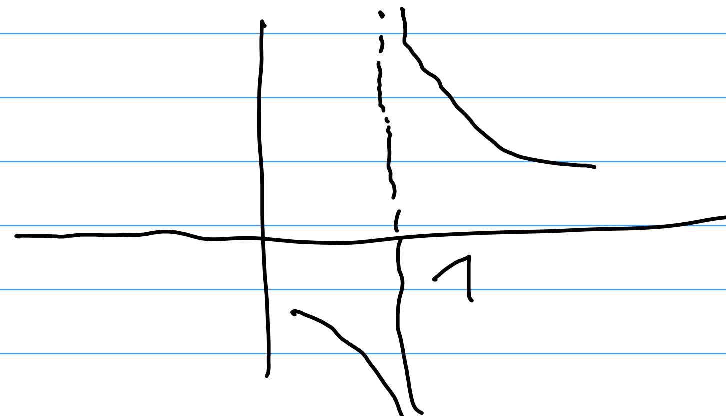
$f(x) = x + \frac{1}{x-1}$ $\qquad D(f) = \mathbb{R} \setminus \{1\}$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \left(x + \frac{1}{x-1}\right) = 1 + \lim_{x \to 1^-} \frac{1}{x-1} =$$

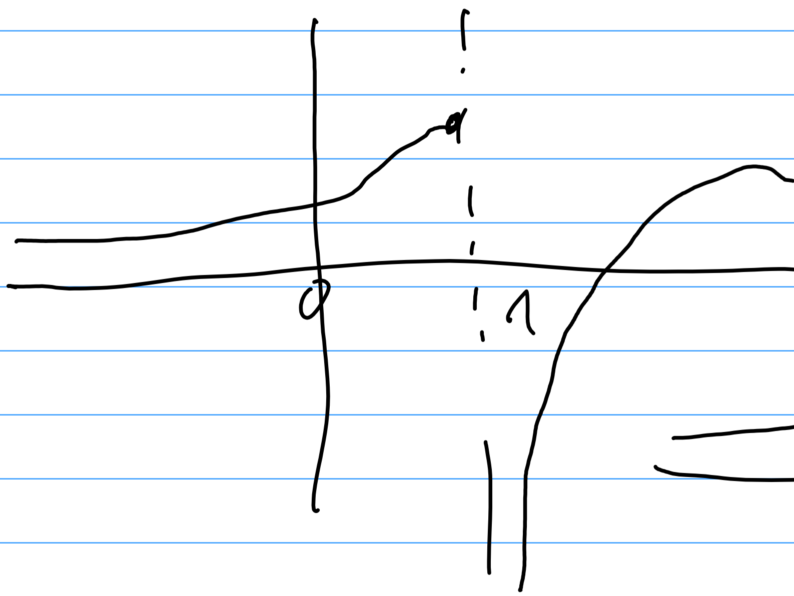
$$= -\infty$$

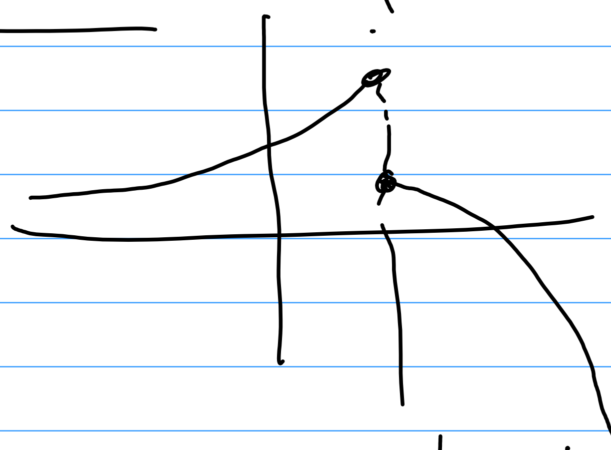
$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \left(x + \frac{1}{x-1}\right) = 1 + \lim_{x \to 1^+} \frac{1}{x-1} =$$

$$= \infty$$



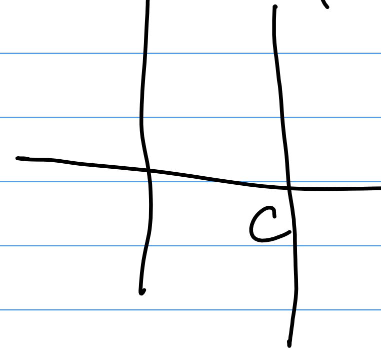
as. bez. sm. $x = 1$





asymptoty so smernicou

bez smernice $x = c$



$$k = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{x + \frac{1}{x-1}}{x} = \lim_{x \to \infty} \left(1 + \frac{1}{x(x-1)}\right) =$$

∞

$$= \lim_{x \to \infty} \left(1 + \frac{1}{x^2} \cdot \frac{1}{1 \cdot \left(1 - \frac{1}{x}\right)}\right)$$

$$= 1 + \lim_{x \to \infty} \frac{1}{x(x-1)} = 1 + 0 = 1$$

$$q = \lim_{x \to \infty} (f(x) - kx) = \lim_{x \to \infty} \left(x + \frac{1}{x-1} - 1 \cdot x\right) =$$

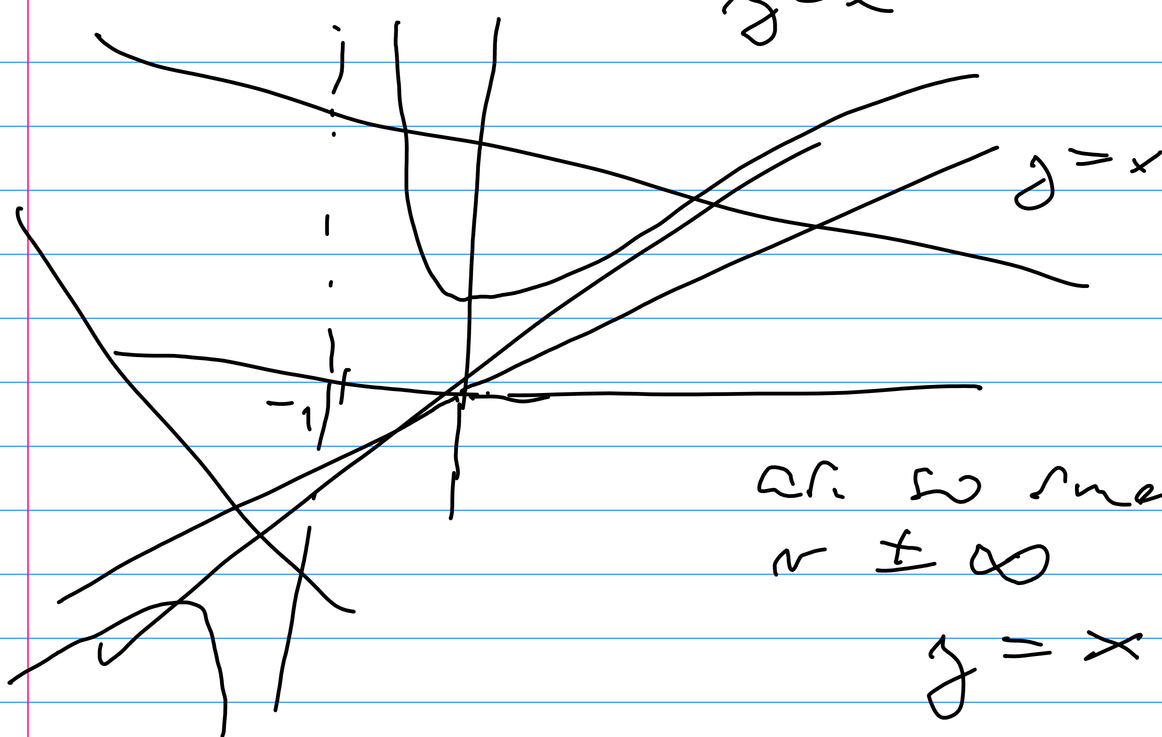
$$= \lim_{x \to \infty} \frac{1}{x-1} = 0$$

v ∞ as. so smer. $y = 1 \cdot x + 0$

$y = x$

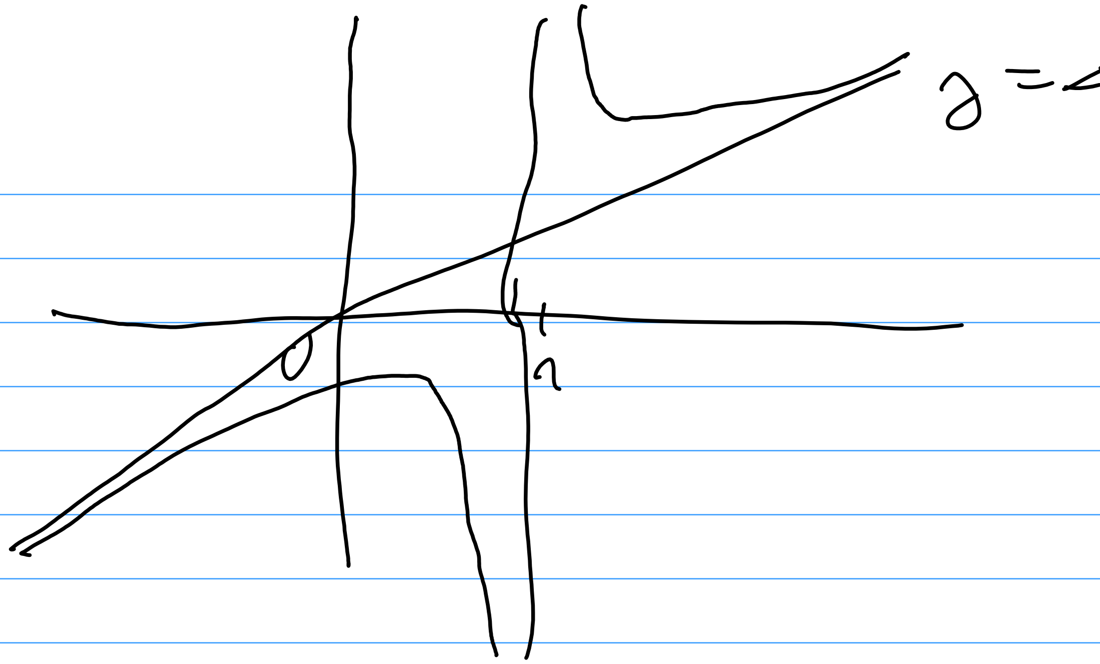
v $-\infty$ as. so smer. $y = 1 \cdot x + 0$

$y = x$



as. so smernicou v $\pm\infty$

$y = x$



$$f(x)=\frac{x^3}{2(x+1)^2}$$

1. definičný obor, body nespojitosti
2. nulové body
3. intervaly monotónnosti, lokálne extrémy
4. konvexnosť, konkávnosť, inflexné body
5. asymptoty
6. graf

1. $D(f)=R\setminus\{-1\}$

   možné body nespojitosti -1

2. $f(x)=0$

   $\frac{x^3}{2(x+1)^2}=0 \qquad x^3=0 \qquad x=0$

nulové body 0

(3.) $$f'(x)=\left[\frac{x^3}{2(x+1)^2}\right]' = \frac{x^2(x+3)}{2(x+1)^3}$$

$f'(x)=0$ stac. body

$x=0$
$x=-3$

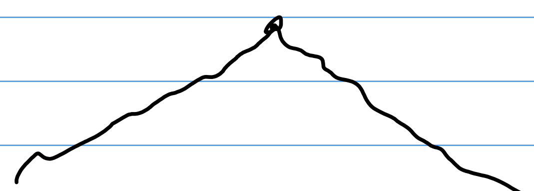
$$\frac{x^2(x+3)}{2(x+1)^3}=0 \qquad x^2(x+3)=0$$

$x^2=0 \vee x+3=0$
$x=0 \vee x=-3$

$-3 \qquad -1 \qquad 0$

	$(-\infty,-3)$	$(-3,-1)$	$(-1,0)$	$(0,\infty)$
f'	+	−	+	+
f	↗	↘	↗	↗

rost. na $(-\infty,-3)$, $(-1,0)$ a na $(0,\infty)$
klés. na $(-3,-1)$
lok. max. -3



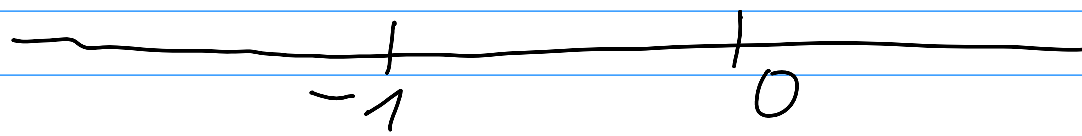
(4) $$f''(x)=\left[\frac{x^2(x+3)}{2(x+1)^3}\right]' = \frac{3x}{(x+1)^4}$$

$$f''(x) = 0 \qquad \frac{3x}{(x+1)^4} = 0$$

$$3x = 0$$

$$x = 0$$

$x = -1$



	$(-\infty,-1)$	$(-1,0)$	$(0,\infty)$
f''	$-$	$-$	$+$
f	$\cap$	$\cap$	$\cup$

konkávna na $(-\infty,-1)$ a na $(-1,0)$

konvexná na $(0,\infty)$

inflexné body $\emptyset$

5. asymptoty bez smernice

$x = -1$

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \frac{x^3}{2(x+1)^2} = -\infty$$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \frac{x^3}{2(x+1)^2} = -\infty$$

$-\infty$

as. bez smernice $x = -1$

as. so smernicou $\qquad y = kx + q$

$$k = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{\frac{x^3}{2(x+1)^2}}{x} =$$

$$= \lim_{x \to \infty} \frac{x^3}{2x(x+1)^2} = \lim_{x \to \infty} \frac{x^2}{2(x+1)^2} =$$

$$= \lim_{x \to \infty} \frac{x^2}{2x^2\left(1+\frac{1}{x}\right)^2} = \lim_{x \to \infty} \frac{1}{2\left(1+\frac{1}{x}\right)^2} = \frac{1}{2}$$

$$q = \lim_{x \to \infty} \left(f(x) - kx\right) = \lim_{x \to \infty} \left[\frac{x^3}{2(x+1)^2} - \frac{1}{2}x\right] =$$

$$-1$$

$$= \lim_{x \to \infty} \frac{x^3 - x(x+1)^2}{2(x+1)^2} =$$

$$= \lim_{x \to \infty} \frac{x^3 - x(x+1)^2}{2(x+1)^2} = \lim_{x \to \infty} \frac{x^3 - x\left(x^2+2x+1\right)}{2(x+1)^2} =$$

$$= \lim_{x \to \infty} \frac{x^3 - x^3 - 2x^2 - x}{2(x+1)^2} = \lim_{x \to \infty} \frac{-2x^2 - x}{2(x+1)^2} =$$

$$= \lim_{x \to \infty} \frac{-2x^2\left(1+\frac{1}{2x}\right)}{2x^2\left(1+\frac{1}{x}\right)^2} = \lim_{x \to \infty} -\frac{1+\frac{1}{2x}}{\left(1+\frac{1}{x}\right)^2} =$$

$$= -1$$

as. so smernicou v ∞ $\quad y = \frac{1}{2}x - 1$

$$(x+1)^2 = \left[x\left(1+\frac{1}{x}\right)\right]^2 = x^2\left(1+\frac{1}{x}\right)^2$$

as. so smernicou v $-\infty$ $\quad y = \frac{1}{2}x - 1$

6.