$$\sum_{n=1}^{\infty} \frac{3n}{4n+1} \qquad \text{diverguje}$$

$$\lim_{n\to\infty} \frac{3n}{4n+1} = \lim_{n\to\infty} \frac{3}{4+\frac{1}{n}} = \frac{3}{4} \neq 0$$

$$\sum_{n=1}^{\infty} a_n \text{ konverguje} \Rightarrow \lim_{n\to\infty} a_n = 0$$

$\Leftarrow$ neplatí

A

$$\lim_{n\to\infty} \frac{1}{n} = 0 \qquad \sum_{n=1}^{\infty} \frac{1}{n} \text{ diverguje}$$

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$$\sum_{n=1}^{\infty} \cos\frac{1}{n^2} \qquad \text{diverguje}$$

$$\lim_{n\to\infty} \cos\frac{1}{n^2} = \cos 0 = 1 \neq 0$$

---

$$\sum_{n=1}^{\infty} \left(\frac{3n}{4n+5}\right)^n \qquad \lim_{n\to\infty} \sqrt[n]{\left(\frac{3n}{4n+5}\right)^n} = \lim_{n\to\infty} \frac{3n}{4n+5} =$$

$$l = \lim_{n\to\infty} \sqrt[n]{a_n} \qquad a_n \geq 0 \qquad = \frac{3}{4} < 1$$

$l < 1$ ... konverguje

$l > 1$ ... diverguje

$l = 1$ ... nevieme

konverguje

Cauchyho krit.

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$$\sum_{n=1}^{\infty} \left(\frac{4n}{3n+1}\right)^{2n}$$

D

K

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{4n}{3n+1}\right)^{2n}} = \lim_{n\to\infty} \left[\left(\frac{4n}{3n+1}\right)^{2n}\right]^{\frac{1}{n}} =$$

$$= \lim_{n\to\infty} \left(\frac{4n}{3n+1}\right)^2 = \left(\frac{4}{3}\right)^2 > 1$$ divergује

Cauchy

---

$$\sum_{n=1}^{\infty} \operatorname{arctg}^n \frac{1}{n}$$

$$\lim_{n\to\infty} \sqrt[n]{c_n} = \lim_{n\to\infty} \sqrt[n]{\operatorname{arctg}^n \frac{1}{n}} = \lim_{n\to\infty} \operatorname{arctg} \frac{1}{n} = \operatorname{arctg} 0 =$$

$$= 0 < 1$$ konvergује

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$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)!}$$

d'Alembert

$$l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

$l < 1$ ... konvergује

$l > 1$ ... divergује

$l = 1$ ... nevíme

K

$$\frac{a_{n+1}}{c_n} = \frac{\frac{1}{(2(n+1)-1)!}}{\frac{1}{(2n-1)!}} = \frac{(2n-1)!}{(2n+1)!} = \frac{(2n-1)!}{(2n+1)(2n)(2n-1)!} =$$

$$= \frac{1}{(2n+1)(2n)}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{c_n} = 0 < 1$$

konvergује

---

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

D

K

$$\frac{a_{n+1}}{a_n}=\frac{\dfrac{[(n+1)!]^2}{(2(n+1))!}}{\dfrac{(n!)^2}{(2n)!}}=\frac{(2(n+1))!\,(n!)^2}{(2n)!\,[(n+1)!]^2}=$$

$= 2n+2)$

$$=\frac{[(n+1)!]^2\,(2n)!}{(n!)^2\,(2n+2)!}=\frac{[(n+1)\,(n!)]^2\,(2n)!}{(n!)^2\,(2n+2)(2n+1)(2n)!}=$$

$$=\frac{(n+1)^2}{(2n+2)(2n+1)}=$$

$\infty$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{(n+1)^2}{(2n+2)(2n+1)}=$$

$$=\lim_{n\to\infty}\frac{n^2\left(1+\frac{1}{n}\right)^2}{n^2\left(2+\frac{2}{n}\right)\left(2+\frac{1}{n}\right)}=\lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)^2}{\left(2+\frac{2}{n}\right)\left(2+\frac{1}{n}\right)}=\frac{1}{4}$$

$\frac{1}{4}<1$ konvergira

d'Alembert

---

$$\sum_{n=1}^{\infty}\frac{4^n}{(n+1)3^n}$$

K

D

$$\frac{a_{n+1}}{a_n}=\frac{\dfrac{4^{n+1}}{(n+2)3^{n+1}}}{\dfrac{4^n}{(n+1)3^n}}=\frac{4^{n+1}(n+1)3^n}{4^n(n+2)3^{n+1}}=\frac{4}{3}\,\frac{n+1}{n+2}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\frac{4}{3}>1$$ divergira

---

$$\sum_{n=1}^{\infty} \frac{2n+3}{n^2+2}$$

$a_n \leq b_n$ $\qquad \sum_{n=1}^{\infty} b_n$ konvergira $\Rightarrow \sum_{n=1}^{\infty} a_n$ konvergira

$\sum_{n=1}^{\infty} a_n$ divergira $\Rightarrow \sum_{n=1}^{\infty} b_n$ div.

$$\sum_{n=1}^{\infty} \frac{2n+3}{n^2+2}$$

$$\underset{\text{div.}}{\frac{2n+3}{n^2+2}} > \frac{2n}{n^2+2} > \frac{2n}{n^2+2n^2} = \frac{2n}{3n^2} = \underset{\text{div.}}{\frac{2}{3} \cdot \frac{1}{n}}$$

---

$$\sum_{n=1}^{\infty} \frac{n-1}{n^3+n}$$

K

D

$$\underset{\text{konv.}}{\frac{n-1}{n^3+n}} < \frac{n}{n^3+n} < \frac{n}{n^3} = \underset{\text{konv}}{\frac{1}{n^2}}$$