$$\sum_{n=1}^{\infty} \frac{3n}{4n+1}$$ nutná podmienka konvergencie **D**

$$\sum_{n=1}^{\infty} a_n \text{ konv} \Rightarrow \lim_{n\to\infty} a_n = 0$$

$$\not\Leftarrow$$

$a_n = \frac{1}{n}$ $\qquad \lim_{n\to\infty} a_n = 0$

$$\sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3n}{4n+1} = \lim_{n\to\infty} \frac{n\cdot 3}{n\left(4+\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{3}{4+\frac{1}{n}}$$

$$= \frac{3}{4} \neq 0$$ diverguje

$$\sum_{n=1}^{\infty} \cos\frac{1}{n^2}$$ **D**

$$\lim_{n\to\infty} \cos\frac{1}{n^2} = \cos 0 = 1 \neq 0$$ diverguje

---

$$\sum_{n=1}^{\infty} \left(\frac{3n}{4n+5}\right)^n \not\Leftarrow \lim_{n\to\infty} a_n = 0$$ **K**

Cauchy

$$l = \lim_{n\to\infty} \sqrt[n]{a_n}$$

$l < 1$ ..... konv.

$l > 1$ ..... div

$l = 1$ ..... nevieme

$$l = \lim_{n\to\infty} \sqrt[n]{\left(\frac{3n}{4n+5}\right)^n} = \lim_{n\to\infty} \frac{3n}{4n+5} = \frac{3}{4} < 1$$

konvergje

$$\sum_{n=1}^{\infty} \left(\frac{4n}{3n+1}\right)^{2n}$$

D

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{4n}{3n+1}\right)^{2n}} = \lim_{n\to\infty} \left(\frac{4n}{3n+1}\right)^{\frac{2n}{n}} =$$

$$= \lim_{n\to\infty} \left(\frac{4n}{3n+1}\right)^2 = \left(\frac{4}{3}\right)^2 > 1$$

divergje

$$\sum_{n=1}^{\infty} \operatorname{arctg}^n \frac{1}{n}$$

K

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\operatorname{arctg}^n \frac{1}{n}} = \lim_{n\to\infty} \operatorname{arctg} \frac{1}{n} =$$

$$= \operatorname{arctg} 0 = 0 < 1$$

Cauchy konvergje

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)!}$$

K

D

N

d'Alembert

$$l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

$l < 1$ .... konvergje

$l > 1$ .... divergje

$l = 1$ .... nevieme

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} =$$

$$= \lim_{n\to\infty} \frac{\frac{1}{(2(n+1)-1)!}}{\frac{1}{(2n-1)!}} = \lim_{n\to\infty} \frac{(2n-1)!}{(2n+1)!} = \lim_{n\to\infty} \frac{(2n-1)!}{(2n+1)(2n)(2n-1)!}$$

$$= \lim_{n\to\infty} \frac{1}{(2n+1)(2n)} = 0 < 1$$ konvergje d'Alembert

$$a_n = \frac{1}{2n-1} \qquad a_{n+1} = \frac{1}{2(n+1)-1}$$

K

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{((n+1)!)^2}{(2(n+1))!}}{\frac{(n!)^2}{(2n)!}} =$$

$$= \lim_{n\to\infty} \frac{((n+1)!)^2 (2n)!}{(n!)^2 (2(n+1))!} = \lim_{n\to\infty} \frac{[(n+1)(n!)]^2 (2n)!}{(n!)^2 (2n+2)(2n+1)(2n)!} =$$

$$= \lim_{n\to\infty} \frac{(n+1)^2 (n!)^2}{(n!)^2 (2n+2)(2n+1)} = \lim_{n\to\infty} \frac{(n+1)^2}{(2n+2)(2n+1)} =$$

$$= \lim_{n\to\infty} \frac{n^2 \left(1+\frac{1}{n}\right)^2}{n^2 \left(2+\frac{2}{n}\right)\left(2+\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{\left(2+\frac{2}{n}\right)\left(2+\frac{1}{n}\right)} =$$

$$= \frac{1}{4} < 1$$ konvergje d'Alembert

D

$$\sum_{n=1}^{\infty} \frac{4^n}{(n+1)3^n}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\dfrac{4^{n+1}}{(n+2)\cdot 3^{n+1}}}{\dfrac{4^n}{(n+1)3^n}} =$$

$$= \lim_{n\to\infty} \frac{4^{n+1}\cdot 3^n (n+1)}{4^n \; 3^{n+1} (n+2)} = \lim_{n\to\infty} \frac{4}{3}\cdot\frac{n+1}{n+2} =$$

$$= \lim_{n\to\infty} \frac{4}{3} \lim_{n\to\infty} \frac{n+1}{n+2} = \frac{4}{3}\cdot 1 = \frac{4}{3} > 1$$

divergije d'Alembert

$$\sum_{n=1}^{\infty} \frac{2n+3}{n^2+2}$$

divergije poređivanje majorantom

$a_n \geq 0 \qquad b_n \geq 0$

$a_n \leq b_n \qquad \sum b_n$ konv. $\Rightarrow \sum a_n$ konv.

$\sum a_n$ div $\Rightarrow \sum b_n$

$$\underline{\frac{2n+3}{n^2+2}} > \frac{2n}{n^2+2} > \frac{2n}{n^2+2n^2} = \frac{2n}{3n^2} = \underline{\frac{2}{3}\cdot\frac{1}{n}}$$

$\sum \frac{1}{n} = \infty$ divergije

$$\sum_{n=1}^{\infty} \frac{n-1}{n^3+n} \qquad \frac{n}{n^3} \qquad \cancel{\frac{1}{n^3}} \quad \frac{1}{n^2}$$

$$\underbrace{\frac{n-1}{n^3+n}}_{\text{konv.}} < \frac{n}{n^3+n} < \frac{n}{n^3} = \underbrace{\frac{1}{n^2}}_{\text{konv.}} \qquad \sum \frac{1}{n^2}$$