$\cos 5^\circ$ $\qquad 10^{-5} = 0{,}00001$

$f(x) = \cos x \qquad x = 5^\circ = 5 \cdot \frac{\pi}{180} = \frac{\pi}{36} < 0{,}1$

střed $\quad a = 0$

$f(x) = \cos x, \quad f'(x) = -\sin x, \quad f''(x) = -\cos x,$

$f'''(x) = \sin x, \quad f^{(4)}(x) = \cos x$

$f(0) = 1, \quad f'(0) = 0, \quad f''(0) = -1, \quad f'''(0) = 0, \quad f^{(4)}(0) = 1$

$n = 3$

$T_3(x) = f(0) + \frac{f'(0)}{1!}(x-0) + \frac{f''(0)}{2!}(x-0)^2 + \frac{f'''(0)}{3!}(x-0)^3 =$

$= 1 + \frac{(-1)}{2!}x^2 = 1 - \frac{1}{2}x^2$

$R_3(x) = f(x) - T_3(x)$

$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^n \qquad c \in \langle a, x\rangle$
$\qquad c \in \langle x, a\rangle$

$R_3(x) = \frac{f^{(4)}(c)}{4!}(x-0)^4 =$

$= \frac{\cos c}{24} x^4 \qquad c \in \langle 0, x\rangle$

$|\cos c| \leq 1$

$|R_3(x)| = \left|\frac{\cos c}{24} x^4\right| \leq \frac{1}{24} x^4$

$\left|R_3\left(\frac{\pi}{36}\right)\right| \leq \frac{1}{24}\left(\frac{\pi}{36}\right)^4 \doteq 2{,}4 \cdot 10^{-6} < 10^{-5}$

přibližně $\quad \cos\frac{\pi}{36} \doteq T_3\left(\frac{\pi}{36}\right) = 1 - \frac{1}{2}\left(\frac{\pi}{36}\right)^2 \doteq 0{,}996192$

presně $\cos\frac{\pi}{36} = 0{,}996194\underline{7}$

$$a_n = \frac{3n+2}{n^2+n+1} \qquad a_2 = \frac{3\cdot 2+2}{2^2+2+1} = \frac{8}{7} \qquad \frac{8}{7}$$

$$a_{k+1} = \frac{3(k+1)+2}{(k+1)^2+k+1+1} = \frac{3k+5}{k^2+3k+3} \qquad \frac{3k+5}{k^2+3k+3}$$

$$1+\frac{2}{3}+\frac{3}{9}+\frac{4}{27}+\dots = \sum_{n=0}^{\infty} a_n \qquad \frac{n}{3n}$$

$$a_n = \frac{n+1}{3^n} \qquad \frac{n}{3^{n-1}} \qquad \frac{n}{3^n}$$

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)} \qquad \frac{A}{(n+1)} + \frac{B}{(n+2)} =$$

$$= \frac{A(n+2)+B(n+1)}{(n+1)(n+2)} = \frac{n(A+B)+2A+B}{(n+1)(n+2)} =$$

$$\begin{aligned} A+B&=0 \\ 2A+B&=1 \end{aligned} \qquad \begin{aligned} A&=1 \\ B&=-1 \end{aligned}$$

$$= \frac{1}{(n+1)(n+2)} = \frac{0n+1}{(n+1)(n+2)}$$

$$\frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2}$$

$$S_N = \left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\dots+\left(\frac{1}{N+1}-\frac{1}{N+2}\right) =$$

$$= \frac{1}{2} - \frac{1}{N+2}$$

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)} = \lim_{N \to \infty} \left(\frac{1}{2} - \frac{1}{N+2} \right) = \frac{1}{2} - 0 = \frac{1}{2}$$

$$\sum_{n=0}^{\infty} 3\left(\frac{1}{4}\right)^n \qquad |q| < 1 \qquad \underline{\underline{4}}$$

$$\sum_{n=0}^{\infty} a q^n \qquad a = 3 \qquad q = \frac{1}{4}$$

$$\sum_{n=0}^{\infty} a q^n = \frac{a}{1-q} = \frac{3}{1-\frac{1}{4}} = \frac{3}{\frac{3}{4}} = \underline{\underline{4}}$$

$$\sum_{n=2}^{\infty} 4\left(-\frac{1}{3}\right)^n \qquad \sum_{n=0}^{\infty} a q^n \qquad 3$$

$$a = \frac{4}{9} = 4\left(-\frac{1}{3}\right)^2 \qquad q = -\frac{1}{3} \qquad |q| < 1$$

$$\sum_{n=2}^{\infty} 4\left(-\frac{1}{3}\right)^n = \frac{a}{1-q} = \frac{\frac{4}{9}}{1-\left(-\frac{1}{3}\right)} = \frac{\frac{4}{9}}{\frac{4}{3}} = \underline{\underline{\frac{1}{3}}}$$

$$\sum_{n=0}^{\infty} \frac{6-2^n}{3^{n+1}} \qquad \frac{6-2^{n+1}}{3^{n+2}} \neq \frac{6-2^n}{3^{n+1}} q$$

$$\sum_{n=0}^{\infty} \frac{6}{3^{n+1}} - \sum_{n=0}^{\infty} \frac{2^n}{3^{n+1}}$$

$$\sum_{n=0}^{\infty} \left[\frac{6}{3^{n+1}} - \frac{2^n}{3^{n+1}} \right] = \sum_{n=0}^{\infty} \left[\frac{6}{3}\left(\frac{1}{3}\right)^n - \frac{1}{3}\left(\frac{2}{3}\right)^n \right]$$

$$\sum_{n=0}^{\infty} a_n + b_n = \sum_{n=0}^{\infty} a_n + \sum_{n=0}^{\infty} b_n$$

$$\sum_{n=0}^{\infty}(a_n + b_n) = \lim_{N\to\infty} \sum_{n=0}^{N}(a_n + b_n) = \lim_{N\to\infty}\left(\sum_{n=0}^{N} a_n + \sum_{n=0}^{N} b_n\right) =$$

$$= \lim_{N\to\infty} S_N^a + S_N^b = S^a + S^b = \sum_{n=0}^{\infty} a_n + \sum_{n=0}^{\infty} b_n$$

$$\sum_{n=0}^{\infty} 2\cdot\left(\frac{1}{3}\right)^n - \sum_{n=0}^{\infty} \frac{1}{3}\left(\frac{2}{3}\right)^n = 3 - 1 = \underline{\underline{2}}$$

$$\sum_{n=0}^{\infty} 2\left(\frac{1}{3}\right)^n = \frac{a}{1-q} = \frac{2}{1-\frac{1}{3}} = \qquad a = 2 \quad q = \frac{1}{3}$$

$$= \frac{2}{\frac{2}{3}} = 3$$

$$\sum_{n=0}^{\infty} \frac{1}{3}\left(\frac{2}{3}\right)^n = \frac{a}{1-q} = \frac{\frac{1}{3}}{1-\frac{2}{3}} = \qquad a = \frac{1}{3} \quad q = \frac{2}{3}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3}} = 1$$

$$\lim_{n\to\infty} \frac{1+2+3+\dots+n}{n} \qquad \infty$$

$$1+2+3+\dots+n = \frac{1}{2}n(n+1)$$

$$\begin{array}{l} 1+2+3+\dots+(n-1)+n \\ \underline{n+(n-1)+(n-2)+\dots\ 2\ \ +1} \\ (n+1)+(n+1)+\dots\dots(n+1)+(n+1) \end{array} \qquad 2S = (n+1)n$$

$$\lim_{n\to\infty} \frac{1+2+3+\dots+n}{n} = \lim_{n\to\infty} \frac{\frac{1}{2}n(n+1)}{n} =$$

$$= \lim_{n\to\infty} \frac{1}{2}(n+1) = \infty$$

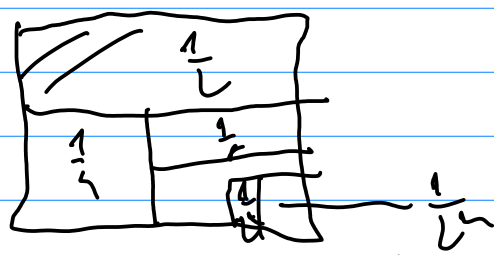
$$\lim_{n\to\infty} \sqrt{2}\,\sqrt[4]{2}\,\sqrt[8]{2}\cdots\sqrt[2^n]{2}$$

∞

0

1

$$a_n = 2^{\frac{1}{2}}\, 2^{\frac{1}{4}}\, 2^{\frac{1}{8}} \cdots 2^{\frac{1}{2^n}} = 2^{\frac{1}{2}+\frac{1}{4}+\dots+\frac{1}{2^n}} = 2^{1-\frac{1}{2^n}}$$



$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} 2^{1-\frac{1}{2^n}} = 2^{\lim\limits_{n\to\infty} 1-\frac{1}{2^n}} = 2^{1-\lim\limits_{n\to\infty}\frac{1}{2^n}} =$$

$$= 2^{1-0} = \underline{\underline{2}}$$