$\cos 5^\circ$ $\qquad\qquad$ $10^{-5} = 0{,}00001$

$f(x) = \cos x, \quad x = 5^\circ = 5 \cdot \frac{\pi}{180} = \frac{\pi}{36} < 0{,}1$

$a = 0$

$f(x) = \cos x, \; f'(x) = -\sin x, \; f''(x) = -\cos x, \; f'''(x) = \sin x$

$f^{(4)} = \cos x$

$f(0) = 1, \; f'(0) = 0, \; f''(0) = -1, \; f'''(0) = 0, \; f^{(4)}(0) = 1$

$T_n(x) \qquad n = 3$

$$T_3(x) = f(0) + \frac{f'(0)}{1!}(x-0) + \frac{f''(0)}{2!}(x-0)^2 + \frac{f'''(0)}{3!}(x-0)^3 =$$

$$= 1 - \frac{1}{2}x^2$$

$$R_3(x) = f(x) - T_3(x)$$

$$R_3(x) = \frac{f^{(4)}(c)}{4!}(x-0)^4 = \qquad c \in \langle 0, x \rangle$$

$$c \in \langle 0; 0{,}1 \rangle$$

$$= \frac{\cos c}{24} x^4$$

$|\cos c| \leq 1 \qquad |R_3(x)| \leq \frac{1}{24} x^4$

$$\left|R_3\left(\frac{\pi}{36}\right)\right| \leq \frac{1}{24}\left(\frac{\pi}{36}\right)^4 \doteq 2{,}4 \cdot 10^{-6} < 10^{-5}$$

přibližně $\quad \cos\frac{\pi}{36} \doteq T_3\left(\frac{\pi}{36}\right) = 1 - \frac{1}{2}\left(\frac{\pi}{36}\right)^2 \doteq 0{,}996192$

přesně $\quad \cos\frac{\pi}{36} = 0{,}996194$

$$a_n = \frac{3n+2}{n^2+n+1} \qquad a_2 = \qquad \frac{8}{7}$$

$$a_2 = \frac{3\cdot 2+2}{2^2+2+1} = \frac{8}{7}$$

$$k \qquad a_{k+1} = \frac{3(k+1)+2}{(k+1)^2+(k+1)+1} = \frac{3k+5}{k^2+3k+3}$$

$$1+\frac{2}{3}+\frac{3}{9}+\frac{4}{27}+\dots = \sum_{n=0}^{\infty} a_n$$

$$\frac{1+n}{3^n} \qquad \cancel{\frac{1}{3^n}}$$

$$\boxed{a_n = \frac{n+1}{3^n}}$$

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)} \qquad \frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2}$$

$$\frac{1}{(n+1)(n+2)} = \frac{A}{n+1} + \frac{B}{n+2}$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+2}\right) = \lim_{k\to\infty} \sum_{n=1}^{k}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$S_k = \sum_{n=1}^{k}\left(\frac{1}{n+1} - \frac{1}{n+2}\right) = \left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\dots+\left(\frac{1}{k+1}-\frac{1}{k+2}\right) =$$

$$=$$

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)} \qquad \frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2}$$

$$S_k = \sum_{n=1}^{k}\left(\frac{1}{n+1} - \frac{1}{n+2}\right) = \left(\frac{1}{2} - \underline{\frac{1}{3}}\right) + \left(\underline{\frac{1}{3}} - \underline{\frac{1}{4}}\right) + \underline{\quad} \dots +$$

$$+\left(\frac{1}{k+1}-\frac{1}{k+2}\right)=\frac{1}{2}-\frac{1}{k+2}$$

$$\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+2)}=\lim_{k\to\infty}s_k=\lim_{k\to\infty}\left(\frac{1}{2}-\frac{1}{k+2}\right)=\frac{1}{2}-0=\frac{1}{2}$$

$$\sum_{n=0}^{\infty}3\left(\frac{1}{4}\right)^n \qquad \sum_{n=0}^{\infty}aq^n \qquad 4$$

$$a=3\left(\frac{1}{4}\right)^0=3 \qquad q=\frac{1}{4}$$

$$|q|<1$$

$$\sum_{n=0}^{\infty}aq^n=\frac{a}{1-q}=\frac{3}{1-\frac{1}{4}}=$$

$$=\frac{3}{\frac{3}{4}}=4$$

$$\sum_{n=2}^{\infty}4\left(-\frac{1}{3}\right)^n \qquad \sum_{n=0}^{\infty}aq^n \qquad a=4\left(-\frac{1}{3}\right)^2=\frac{4}{9}$$

$$|q|=\left|-\frac{1}{3}\right|=\frac{1}{3}<1 \qquad q=-\frac{1}{3}$$

$$\sum_{n=2}^{\infty}4\left(-\frac{1}{3}\right)^n=\sum_{n=0}^{\infty}\frac{4}{9}\left(-\frac{1}{3}\right)^n=\frac{a}{1-q}=\frac{\frac{4}{9}}{1-\left(-\frac{1}{3}\right)}=$$

$$=\sum_{n=0}^{\infty}aq^n$$

$$=\frac{\frac{4}{9}}{\frac{4}{3}}=\frac{1}{3}$$

$$\sum_{n=0}^{\infty}\frac{6-2^n}{3^{n+1}}=\sum_{n=0}^{\infty}\left(\frac{6}{3^{n+1}}-\frac{2^n}{3^{n+1}}\right)=\sum_{n=0}^{\infty}\frac{6}{3^{n+1}}-\sum_{n=0}^{\infty}\frac{2^n}{3^{n+1}}$$

$$\sum_{n=0}^{\infty}(a_n+b_n) = \underbrace{\sum_{n=0}^{\infty} a_n}\ + \sum_{n=0}^{\infty} b_n$$

$$\sum_{n=0}^{\infty} a_n \qquad \sum_{n=0}^{\infty} b_n$$

$$\sum_{n=0}^{\infty}(a_n+b_n) = \lim_{k\to\infty}\sum_{n=0}^{k}(a_n+b_n) =$$

$$= \lim_{k\to\infty}\left(\sum_{n=0}^{k} a_n + \sum_{n=0}^{k} b_n\right) = \lim_{k\to\infty}\sum_{n=0}^{k} a_n + \lim_{k\to\infty}\sum_{n=0}^{k} b_n =$$

$$= \sum_{n=0}^{\infty} a_n + \sum_{n=0}^{\infty} b_n$$

$$\sum_{n=0}^{\infty}\frac{6}{3^{n+1}} \qquad q=\frac{1}{3} \qquad a=\frac{6}{3^{0+1}}=\frac{6}{3}=2$$

$$\sum = \sum_{n=0}^{\infty} 2\cdot\left(\frac{1}{3}\right)^n = \frac{a}{1-q} = \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}} = 3$$

$$\sum_{n=0}^{\infty}\frac{2^n}{3^{n+1}} \qquad q=\frac{2}{3} \qquad a=\frac{1}{3}$$

$$|q| = \frac{2}{3} < 1$$

$$\sum_{n=0}^{\infty}\frac{1}{3}\left(\frac{2}{3}\right)^n = \frac{a}{1-q} = \frac{\frac{1}{3}}{1-\frac{2}{3}} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$$

$$\sum_{n=0}^{\infty}\frac{6-2^n}{3^{n+1}} = \sum_{n=0}^{\infty}\frac{6}{3^{n+1}} - \sum_{n=0}^{\infty}\frac{2^n}{3^{n+1}} = 3-1 = \underline{\underline{2}}$$

$$\lim_{n\to\infty} \frac{1+2+3+\dots+n}{n}$$

$$1+2+3+\dots+n = \sum_{k=1}^{n} k$$

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

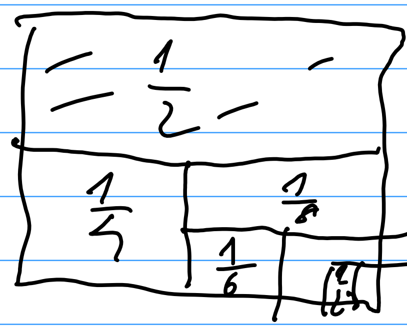
$$\lim_{n\to\infty} \frac{1+2+3+\dots+n}{n} = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{n} =$$

$$= \lim_{n\to\infty} \frac{n+1}{2} = \infty$$

$$\lim_{n\to\infty} \sqrt{2}\,\sqrt[4]{2}\,\sqrt[8]{2}\cdots\sqrt[2^n]{2} = 2$$

$$a_n = \sqrt{2}\,\sqrt[4]{2}\,\sqrt[8]{2}\cdots\sqrt[2^n]{2} = 2^{\frac{1}{2}}\,2^{\frac{1}{4}}\,2^{\frac{1}{8}}\cdots 2^{\frac{1}{2^n}} =$$

$$= 2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots+\frac{1}{2^n}}$$



$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots+\frac{1}{2^n} = 1-\frac{1}{2^n}$$

$$a_n = 2^{1-\frac{1}{2^n}}$$

$$\lim_{n\to\infty} a_n = 2^{1-\lim\limits_{n\to\infty}\frac{1}{2^n}} = 2^{1-0} = 2$$