9 Asymptoty.

Príklad 1. Vyšetrite asymptoty funkcie

$$f(x) = 3x + \frac{1}{x^2}$$

v $\pm\infty$ (ak existujú).

Riešenie.

Počítajme limity

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{3x + \frac{1}{x^2}}{x} = 3,$$

$$q = \lim_{x\to\infty} f(x) - kx = \lim_{x\to\infty} 3x + \frac{1}{x^2} - 3x = 0.$$

Preto priamka $y = 3x$ je asymptota funkcie f v $+\infty$.
Podobne

$$k = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{3x + \frac{1}{x^2}}{x} = 3,$$

$$q = \lim_{x\to-\infty} f(x) - kx = \lim_{x\to-\infty} 3x + \frac{1}{x^2} - 3x = 0.$$

Priamka $y = 3x$ je asymptota funkcie f v $-\infty$.

Príklad 2. Vyšetrite asymptoty funkcie

$$f(x) = \frac{x^2}{\sqrt{4x^2+3}}$$

v $\pm\infty$ (ak existujú).

Riešenie.

Počítajme limity

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\frac{x^2}{\sqrt{4x^2+3}}}{x} = \lim_{x\to\infty} \frac{x}{\sqrt{4x^2+3}} = \lim_{x\to\infty} \frac{x}{x\sqrt{4+\frac{3}{x^2}}} = \frac{1}{2},$$

$$q = \lim_{x\to\infty} f(x) - kx = \lim_{x\to\infty} \frac{x^2}{\sqrt{4x^2+3}} - \frac{1}{2}x = \lim_{x\to\infty} \frac{2x^2 - x\sqrt{4x^2+3}}{2\sqrt{4x^2+3}} =$$

$$= \lim_{x\to\infty} \frac{2x - \sqrt{4x^2+3}}{2\sqrt{1+\frac{3}{4x^2}}} = \lim_{x\to\infty} \frac{-3}{2(2x+\sqrt{4x^2+3})\sqrt{1+\frac{3}{4x^2}}} = 0.$$

Preto priamka $y = \frac{1}{2}x$ je asymptota funkcie f v $+\infty$.
Podobne

$$k = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{\frac{x^2}{\sqrt{4x^2+3}}}{x} = \lim_{x\to-\infty} \frac{x}{\sqrt{4x^2+3}} = \lim_{x\to-\infty} \frac{x}{|x|\sqrt{4+\frac{3}{x^2}}} = -\frac{1}{2},$$

$$q = \lim_{x\to-\infty} f(x) - kx = \lim_{x\to-\infty} \frac{x^2}{\sqrt{4x^2+3}} + \frac{1}{2}x = \lim_{x\to-\infty} \frac{2x^2 - x\sqrt{4x^2+3}}{2\sqrt{4x^2+3}} =$$

$$= \lim_{x\to-\infty} \frac{2x - \sqrt{4x^2+3}}{-2\sqrt{1+\frac{3}{4x^2}}} = \lim_{x\to-\infty} \frac{-3}{-2(2x+\sqrt{4x^2+3})\sqrt{1+\frac{3}{4x^2}}} = 0.$$

Priamka $y = -\frac{1}{2}x$ je asymptota funkcie f v $-\infty$.