1. Riešte systémy lineárnych rovníc.

a.
$$\begin{aligned} x_1 + 2x_2 &= -3 \\ 3x_2 &= -6 \end{aligned}$$

b.
$$\begin{aligned} 3x_1 + x_2 + 2x_3 &= 11 \\ -3x_2 + x_3 &= -3 \\ 7x_3 &= 21 \end{aligned}$$

c.
$$\begin{aligned} 3x_1 + 2x_2 + 3x_3 &= 5 \\ -x_2 + x_3 &= i \\ 2x_3 &= 2 + 2i \end{aligned}$$

d.
$$\begin{aligned} 2x_1 + x_2 - 3x_3 &= -4 \\ x_2 - x_3 &= 2 \end{aligned}$$

2. Napíšte sústavu lineárnych rovníc a množinu všetkých jej riešení, ak jej rozšírená matica je

a. $\left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right)$ b. $\left(\begin{array}{cccc|c} 2 & -1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{array}\right)$

c. $\left(\begin{array}{ccc|c} 1+i & 1 & 1 & 1 \\ 0 & i & 2 & i \\ 0 & 0 & 1 & 1+i \end{array}\right)$ d. $\left(\begin{array}{cccc|c} 0 & -1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right)$

e. $\left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right)$ f. $\left(\begin{array}{cccc|c} 2 & -1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{array}\right)$

3. Rozhodnite, či je daná matica stupňovitá.

a. $\begin{pmatrix} 0 & 2 & -1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ b. $\begin{pmatrix} 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$ c. $\begin{pmatrix} 3 & 2 & -2 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 20 \end{pmatrix}$

4. Riešte sústavy lineárnych rovníc

a.
$$\begin{aligned} x_1 - x_2 &= -2 \\ -3x_1 + 2x_2 &= 3 \end{aligned}$$

b.
$$\begin{aligned} 12x_1 - x_2 + 5x_3 &= 30 \\ 3x_1 - 13x_2 + 2x_3 &= 21 \\ 7x_1 + 2x_2 + 3x_3 &= 15 \end{aligned}$$

c.
$$\begin{aligned} 7x_1 + 3x_2 - 2x_3 &= 1 \\ -x_1 + 6x_2 - 3x_3 &= 2 \\ -10x_1 + 15x_2 - 11x_3 &= 4 \end{aligned}$$

d.
$$\begin{aligned} 2x_1 - x_2 - 4x_3 &= 1 \\ x_1 - x_2 + x_3 &= 2 \\ 4x_1 - x_2 - 2x_3 &= 5 \end{aligned}$$

e.
$$\begin{aligned} 3x_1 - x_2 - 6x_3 - 4x_4 &= 2 \\ 3x_1 + 4x_2 + 3x_3 - 2x_4 &= 12 \\ 4x_1 + 3x_2 - 3x_3 + 4x_4 &= -1 \\ 5x_1 + 4x_2 - 9x_3 &= 5 \end{aligned}$$

f.
$$\begin{aligned} 2x_1 + 2x_2 + 2x_3 - 6x_4 &= 1 \\ 2x_1 + 2x_2 - 6x_3 + 2x_4 &= 2 \\ 2x_1 - 6x_2 + 2x_3 + 2x_4 &= 3 \\ -6x_1 + 2x_2 + 2x_3 + 2x_4 &= 4 \end{aligned}$$

g.
$$\begin{aligned} 2x_1 + (2-i)\,x_2 &= 9 \\ -x_1 + x_2 &= i \end{aligned}$$

h.
$$\begin{aligned} 3x_1 + 2x_2 + x_2 &= 2 + i \\ x_1 + 4x_2 + 7x_3 &= 14 - 3i \\ x_1 + 3x_2 + 4x_3 &= 8 - 2i \end{aligned}$$

5. Riešte dva systémy s rovnakou maticou pomocou eliminácie na matici 3×5.

$$\begin{aligned} x_1 + 2x_2 - 2x_3 &= 1 \\ 2x_1 + 5x_2 + x_3 &= 9 \\ x_1 + 3x_2 + 4x_3 &= 9 \end{aligned} \qquad \begin{aligned} x_1 + 2x_2 - 2x_3 &= 9 \\ 2x_1 + 5x_2 + x_3 &= 9 \\ x_1 + 3x_2 + 4x_3 &= -2 \end{aligned}$$

6. Riešte homogénne sústavy lineárnych rovníc

a.
$$\begin{aligned} 2x_1 - x_2 + 3x_3 + x_4 &= 0 \\ x_1 - x_2 + x_3 + x_4 &= 0 \\ 3x_1 + x_2 + 2x_3 &= 0 \end{aligned}$$

b.
$$\begin{aligned} 2x_1 - x_2 + 3x_3 + x_4 &= 0 \\ x_1 - x_2 + x_3 + x_4 &= 0 \\ 3x_1 + x_2 + 2x_3 &= 0 \\ x_1 + x_2 + x_3 + x_4 &= 0 \end{aligned}$$

7. Riešte sústavy s parametrom $\lambda \in R$

a.
$$\begin{aligned} x + \lambda y &= 1 \\ x + \lambda^2 y &= 1 \end{aligned}$$

b.
$$\begin{aligned} x + \lambda y &= 1 \\ \lambda x + y &= 1 \end{aligned}$$

c.
$$\begin{aligned} x_1 + x_2 + \lambda x_3 &= 1 \\ x_1 + \lambda x_2 + x_3 &= 1 \\ \lambda x_1 + x_2 + x_3 &= \lambda \end{aligned}$$

d.
$$\begin{aligned} x_1 + x_2 + \lambda x_3 &= 1 \\ x_1 + \lambda x_2 + x_3 &= \lambda \\ \lambda x_1 + x_2 + x_3 &= \lambda^2 \end{aligned}$$

Výsledky

1. a) $(1, -2)$, b) $(1, 2, 3)$, c) $(-i, 1, 1+i)$
   d) $\{(-3+a, 2+a, a) \colon a \in R\}$
2. a) $\{(2+p, p, 0, -1) \colon p \in R\}$, b) $\emptyset$,
   c) $(-1-2i, -1+2i, 1+i)$, d) $\{(a, -2, 0, -1) \colon a \in R\}$,
   e) $\{(1+b-a, b, 0, a) \colon a, b \in R\}$,
   f) $\left\{\left(\frac{1-3p-q}{2}, 1-p, p, q\right) \colon p, q \in R\right\}$
3. a) áno, b) nie, c) áno
4. a) $(1, 3)$, b) $(2, -1, 1)$, c) $\left(\frac{1}{60}, \frac{83}{180}, \frac{1}{4}\right)$
   d) $\left(\frac{3}{2}, 0, \frac{1}{2}\right)$, e) $\left(0, 2, \frac{1}{3} - \frac{3}{2}\right)$, f) $\emptyset$, g) $(2, 2+i)$, h) $(i, -i, 2)$
5. $(-1, 2, 1)$, $(3, 1, -2)$
6. a) $\{a(-2, 4, 1, 5) \colon a \in R\}$, b) $\{(0, 0, 0, 0)\}$
7.

a) $\mathcal{R}_{\lambda=0} = \{(1, p) \colon p \in R\}$, $\mathcal{R}_{\lambda=1} = \{(1-p, p) \colon p \in R\}$, $\mathcal{R}_{\lambda \notin \{0,1\}} = \{(1, 0)\}$

b) $\mathcal{R}_{\lambda=1} = \{(1-p, p) \colon p \in R\}$, $\mathcal{R}_{\lambda=-1} = \emptyset$, $R_{\lambda \notin \{1,-1\}} = \{(\frac{1}{1+\lambda}, \frac{1}{1+\lambda})\}$

c) $\mathcal{R}_{\lambda=1} = \{1-p-q, p, q) \colon p, q \in R\}$, $\mathcal{R}_{\lambda=-2} = \{3-p, p, p) \colon p \in R\}$,
$\mathcal{R}_{\lambda \notin \{1,-2\}} = (1, 0, 0)\}$

c) $R_{\lambda=1} = \{1-p-q, p, q) \colon p, q \in R\}$, $\mathcal{R}_{\lambda=-2} = \emptyset$
$R_{\lambda \notin \{1,-2\}} = \left\{\left(\frac{(\lambda+1)^2}{\lambda+2}\right), \frac{1}{\lambda+2}, -\frac{\lambda+1}{\lambda+2}\right)\right\}$